Physical events and quantum field theory without Higgs

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Abstract

All physics events are expressed by particles which similar to well-known elementary particles - leptons, quarks and gauge bosons. Higgs is not necessary.

1 Denotations:

c = 1: the light velocity in vacuum;

$\alpha \cap \beta$: conjunction of events α and β: "α and β";

$\alpha \cup \beta$: disjunction of events α and β: "α or/and β";

$\overline{\alpha}$: event, complementary to event α: "not α";

$\forall$: generality quantifier: "for every";

$\exists$: existenial quantifier: "for some";

$\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are the Cartesian basis vectors;

$\mathbf{x} \overset{Def}{=} (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3)$;

$x_0 \overset{Def}{=} t$;

$\int d^3\mathbf{x} \overset{Def}{=} \int dx_1 \int dx_2 \int dx_3$;

$\partial_k \overset{Def}{=} \partial/\partial x_k$;

$\partial_t \overset{Def}{=} \partial_0 \overset{Def}{=} \partial/\partial t$;

$\partial'_k \overset{Def}{=} \partial/\partial x'_k$;

$$\sum_{\mathbf{k}} \overset{Def}{=} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \sum_{k_3=-\infty}^{\infty};$$

$$1_2 \overset{Def}{=} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, 0_2 \overset{Def}{=} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\beta^{[0]} \overset{Def}{=} -1_4 \overset{Def}{=} -\begin{bmatrix} 1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, 1_8 \overset{Def}{=} \begin{bmatrix} 1_4 & 0_4 \\ 0_4 & 1_4 \end{bmatrix},$$

$$\gamma^{[5]} \overset{Def}{=} \begin{bmatrix} 1_2 & 0_2 \\ 0_2 & -1_2 \end{bmatrix}, 0_4 \overset{Def}{=} \begin{bmatrix} 0_2 & 0_2 \\ 0_2 & 0_2 \end{bmatrix},$$

the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

2 Introduction

I call event, occured in single point of space-time, as *a point event*. And event, formed by the point events, is called as *a physics event*.

Obviously, the double-slits experiment proves that an elementary particle does not exist continuously, but such particle is a set of the point events, bounded by probabilities [1], [2], [3], [4], [5].

These probabilities are expressed by the spinor functions and by *operators of a probability creation* and *a probability annihilation*. These operators are similar to the field operators of QFT. The motion equations in form of the Dirac equations with the additional fields are obtained for the spinor functions. Some of these additional fields behave as the mass members, and other ones behave as the gauge fields.

A 3+1 vector of a probability current and *3-vectors of a average velocity* and of *a local velocity for the probability propagation* are defined by these motion equations.

A set $\widetilde{C}$ of complex $n \times n$ matrices is called as *Clifford's set* [10] if the following conditions are fulfilled:

if $\alpha_k \in \widetilde{C}$ and $\alpha_r \in \widetilde{C}$ then $\alpha_k\alpha_r + \alpha_r\alpha_k = 2\delta_{k,r}$;

if $\alpha_k\alpha_r + \alpha_r\alpha_k = 2\delta_{k,r}$ for all elements α_r of set $\widetilde{C}$ then $\alpha_k \in \widetilde{C}$.

If $n = 4$ then the Clifford's set either contains 3 matrices (*Clifford's triplet*) or contains 5 matrices (*Clifford's pentad*).

6 Clifford's pentads exists, only [10]:
one *light pentad* β:

$$\beta^{[1]} \stackrel{Def}{=} \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \beta^{[2]} \stackrel{Def}{=} \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \beta^{[3]} \stackrel{Def}{=} \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix}, \tag{1}$$

$$\gamma^{[0]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}, \tag{2}$$

$$\beta^{[4]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}; \tag{3}$$

three *chromatic* pentads (see Appendix): *red, blue* and *green* ones; and two *taste* pentads: *sweet* and *bitter* pentads.

At first the motion equations, held the light pentad elements, only, are considered. Such equations are called as equations for *a leptonn* motion.

The Dirac equation contains four elements of Clifford's pentads, only. Three of these elements (1) accord to three space coordinates, and fourth element (2) either constitute the mass member or accords to the time coordinate. But Clifford's pentad holds five elements. Certainly fifth element (3) of the pentad should be added to the motion equation. That is the Dirac equation mass part will hold two members. Moreover, if two additional quasi-space coordinates shall be put in accordance to these two Clifford's pentads mass elements ((2) and (3)) then the homogeheous Dirac equation will be obtained. All five elements of Clifford's pentads and all five space coordinates are held alike in this equation. All local velocities magnitudes equal to unit (c) in such five- dimensional space.

The overdefinited by the such ways equation of motion is invariant for a rotation in 2-space of fourth and fifth coordinates. This transformation defines a field, similar to B-boson field.

Since values of probability are not defined absolutely exactly then the forming mass members additional fields are expressible by sufficiently fine stratums. And values of this two spatial coordinates, added for homogeneity of mass members, can be delimitted by a large number without loss of generality. In this case a masses spectrum is got discrete, and for every 3+1 space-time points: either the single mass is put in this point or this point is empty.

A mass is expressed by the 2-root from sum of two quadrates of integer numbers from two mass members of the motion equation:

$$m_0 = \sqrt{n_0^2 + s_0^2}.$$

But this motion equation is invariant for rotations in 2-space of fourth and fifth coordinates. That is a mss must be expressed by natural number, and numbers of the mass members must remain integer in these rotations.

Hence a mass and the making this mass members numbers should be Pythagorean triplet $\langle m_0; n_0, s_0 \rangle$ [7]. Here m_0 is called as *a triplet father*. One triplet substitutes another triplet with the same father at rotations.

Let $\jmath$ ($\jmath = \{\alpha_0, \alpha_1, \ldots, \alpha_n\}$) be a finite set of angles (positive real numbers) such that

$$\sum_{k=0}^{n} \alpha_k = 2\pi$$

and

$$\varepsilon = \max\left(\alpha_0, \alpha_1, \ldots, \alpha_n\right)$$

with ε as a tiny real positive number.

In that case $\jmath$ is called as 2π *coat with precision* ε.

Let m be an natural number such that a Pythagorean triplet $\langle m; n, s \rangle$ exists for every element α_k of $\jmath$ with

$$n = m \sin \alpha_k,$$
$$s = m \cos \alpha_k.$$

In that case m is called as *a father of* $\jmath$.

At far areas of the natural number array the 2π coats fathers exist for any high precision.

I believe that these families of the Pythagorean triplets conform to the families of elementary particles.

The *particles creation* and *the particles annihilation operators* are denoted as the Fourier transformations of corresponding operators for probability, and antiparticles are denoted by the standard way.

Subsequently here are considered all unitary transformations on the two-masses functions such that these transformations retain the probability 4-vectors. The transformations, adequate to electroweak gauge fields, exist

among these unitary transformations. These electroweak unitary transformations are expressed by rotations in 2-space of fourth and fifth coordinates, too. Particles, similar to neutrino (*neutrinno*), arise in these transformations. Neutrinnos prove to be connected essentially with their *leptonns.*

The motions equations is invariant for these transformations, and fields, similar to the boson ones, arise as result. The massless field

$$W_{\mu,\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - \mathrm{i}\frac{g_2}{2}\left(W_\mu W_\nu - W_\nu W_\mu\right)$$

denoted by usual way, but the motion equations for fields W_μ are similar to Klein-Gordon equation with nonzero mass.

Massless field A and the massive field Z are denoted by standard way by fields B and W.

The leptonn motion equation is invariant for rotations in 3-space of first usual spatial coordinates and for Lorentz's rotations in 3+1 space-time. The motion equations, maked by the chromatic pentads, are mixed between each other. That is particles, conformed to pentads of different colors, is unseparable in space-time (confinement). Two sorts of three colors of such particles are held in one family. Hence - six. I call these particles as *quarrks.*

3 Events

Let Ω_0 be a finite set of $N^{\text{`}}$ point-events $A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right)$ ($N^{\text{`}}$ is natural number, ι is natural index number).

Let Ω be set of events for which:

1) $\Omega_0 \subseteq \Omega$;

2) if $B \in \Omega$ and $C \in \Omega$ then $(B \cup C) \in \Omega$ and $(B \cap C) \in \Omega$;

3) if $B \in \Omega$ then $\overline{B} \in \Omega$;

4) if $A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right) \in \Omega$ then

$\left(\forall \mathbf{x}^{(\iota)} : A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right)\right) \in \Omega$ and $\left(\exists \mathbf{x}^{(\iota)} : A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right)\right) \in \Omega$.

I call elements of Ω as *a physics events.*

Let:

$$\alpha_{\iota_1,\iota_2,\ldots,\iota_s}\left(t, \mathbf{x}^{(\iota_1)}, \mathbf{x}^{(\iota_2)}, \ldots, \mathbf{x}^{(\iota_s)}\right) \stackrel{Def}{=}$$

$$\stackrel{Def}{=} \left(\begin{array}{c} \left(A_{\iota_1}\left(t, \mathbf{x}^{(\iota_1)}\right) \cap A_{\iota_2}\left(t, \mathbf{x}^{(\iota_2)}\right) \cap \cdots \cap A_{\iota_s}\left(t, \mathbf{x}^{(\iota_s)}\right)\right) \cap \\ \left(\bigcap_{\iota \notin \{\iota_1,\iota_2,\ldots,\iota_s\}}^{N^{\text{`}}} \left(\forall \mathbf{x}^{(\iota)} : \overline{A_\iota\left(t, \mathbf{x}^{(\iota)}\right)}\right)\right) \end{array} \right).$$

For example:

$$\alpha_0(t) \stackrel{Def}{=} \cap_{\iota=1}^{N^\iota} \forall \mathbf{x}^{(\iota)} : \overline{A_\iota(t, \mathbf{x}^{(\iota)})};$$

$$\alpha_1\left(t, \mathbf{x}^{(1)}\right) \stackrel{Def}{=}$$
$$\stackrel{Def}{=} A_1\left(t, \mathbf{x}^{(1)}\right) \cap \left(\cap_{\iota \neq 1}^{N^\iota} \forall \mathbf{x}^{(\iota)} : \overline{A_\iota(t, \mathbf{x}^{(\iota)})}\right);$$

$$\alpha_{1,2}\left(t, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}\right) \stackrel{Def}{=}$$

$$\stackrel{Def}{=} \left(A_1\left(t, \mathbf{x}^{(1)}\right) \cap A_2\left(t, \mathbf{x}^{(2)}\right)\right) \cap \left(\cap_{\iota \notin \{1,2\}}^{N^\iota} \forall \mathbf{x}^{(\iota)} : \overline{A_\iota(t, \mathbf{x}^{(\iota)})}\right).$$

Let $\mathbf{P}$ be a probability function, defined on Ω.
Because $A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right)$ is a point-event then for $m \neq n$:

$$\mathbf{P}\left(A_\iota\left(t_m^{(\iota)}, \mathbf{x}_m^{(\iota)}\right) \cap A_\iota\left(t_n^{(\iota)}, \mathbf{x}_n^{(\iota)}\right)\right) = 0.$$

By idempotence:

$$\mathbf{P}\left(A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right) \cap A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right)\right) = \mathbf{P}\left(A_\iota\left(t^{(\iota)}, \mathbf{x}^{(\iota)}\right)\right)$$

Let $\mathbf{H}$ be a Hilbert space.
Let linear operators $\psi_j^{(\iota)}\left(\mathbf{x}^{(\iota)}\right)$ ($j \in \{1,2,3,4\}$) act on elements of $\mathbf{H}$. And these operators have got the following properties:

1. $\mathbf{H}$ contains the element Φ_0 for which:

$$\Phi_0^\dagger \Phi_0 = 1$$

and

$$\psi_j^{(\iota)} \Phi_0 = 0,\ \Phi_0^\dagger \psi_j^{(\iota)\dagger} = 0;$$

2.

$$\psi_j^{(\iota)}\left(\mathbf{x}^{(\iota)}\right) \psi_j^{(\iota)}\left(\mathbf{x}^{(\iota)}\right) = 0$$

and

$$\psi_j^{(\iota)\dagger}\left(\mathbf{x}^{(\iota)}\right)\psi_j^{(\iota)\dagger}\left(\mathbf{x}^{(\iota)}\right)=0;$$

3.

$$\begin{gathered}\left\{\psi_{j'}^{(\iota')\dagger}\left(\mathbf{y}^{(\iota')}\right),\psi_j^{(\iota)}\left(\mathbf{x}^{(\iota)}\right)\right\}\stackrel{Def}{=}\\ \stackrel{Def}{=}\psi_{j'}^{(\iota')\dagger}\left(\mathbf{y}^{(\iota')}\right)\psi_j^{(\iota)}\left(\mathbf{x}^{(\iota)}\right)+\psi_j^{(\iota)}\left(\mathbf{x}^{(\iota)}\right)\psi_{j'}^{(\iota')\dagger}\left(\mathbf{y}^{(\iota')}\right)\\ =\delta_{\iota',\iota}\delta\left(\mathbf{y}^{(\iota')}-\mathbf{x}^{(\iota)}\right)\delta_{j',j}\end{gathered} \tag{4}$$

I call operator $\psi^{(\iota)\dagger}(\mathbf{x})$ as *a creation operator* and $\psi^{(\iota)}(\mathbf{x})$ -as *an annihilation operator of the event A_ι probability at point* $\mathbf{x}$.

4 An one event case

4.1 Hamiltonians

Let $N` = 1$.

Let real function $\rho(t,\mathbf{x})$ be *the probability density of the event* $\alpha_1(t,\mathbf{x})$. That is for each domain D ($D \subseteq R^3$):

$$\int_D d^3\mathbf{x}\cdot\rho(t,\mathbf{x})\stackrel{Def}{=}\mathbf{P}\left(\exists\mathbf{x}\in D:\alpha_1(t,\mathbf{x})\right).$$

Complex functions $\varphi_j(t,\mathbf{x})$ ($j\in\{1,2,3,4\}$) exist such that

$$\rho(t,\mathbf{x})=\sum_{j=1}^{4}\varphi_j^*(t,\mathbf{x})\varphi_j(t,\mathbf{x}). \tag{5}$$

Let us denote $\Psi(t,\mathbf{x})$ as the following:

$$\Psi(t,\mathbf{x})\stackrel{Def}{=}\sum_{j=1}^{4}\varphi_j(t,\mathbf{x})\psi_j^\dagger(\mathbf{x})\Phi_0 \tag{6}$$

From (4):

$$\Psi^\dagger(t,\mathbf{x}')\Psi(t,\mathbf{x})=\sum_{j=1}^{4}\varphi_j^*(t,\mathbf{x}')\varphi_j(t,\mathbf{x})\delta(\mathbf{x}'-\mathbf{x}).$$

I consider the events, fulfilled to the following condition, only: there exists a tiny real positive number h such that if $|x_r|\geq\frac{\pi}{h}$ ($r\in\{1,2,3\}$) then

$$\varphi_j(t,\mathbf{x}) = 0.$$

Let (V) be denoted as the following: $\mathbf{x} \in (V)$ if and if, only, $|x_r| \leq \frac{\pi}{h}$ for $r \in \{1,2,3\}$. That is:

$$\int_{(V)} d^3\mathbf{x} = \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} dx_1 \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} dx_2 \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} dx_3.$$

If

$$\overline{\mathbf{x}}(t) = \int d^3\mathbf{x} \cdot \mathbf{x}\rho(t,\mathbf{x})$$

then $\overline{\mathbf{x}}(t)$ is *the average coordinate of the event* α_1 and:

$$\overline{\mathbf{x}}(t) = \int d^3\mathbf{x} \cdot \mathbf{x} \sum_{j=1}^{4} \varphi_j^*(t,\mathbf{x})\, \varphi_j(t,\mathbf{x}),$$

and *the average velocity of the event* α_1 *probability* is:

$$\begin{array}{c} \overline{\mathbf{v}}(t) = \partial_t \overline{\mathbf{x}}(t) = \\ = \int d^3\mathbf{x} \cdot \mathbf{x} \sum_{j=1}^{4} \left(\begin{array}{c} \left(\partial_t \varphi_j^*(t,\mathbf{x})\right) \varphi_j(t,\mathbf{x}) \\ +\varphi_j^*(t,\mathbf{x}) \left(\partial_t \varphi_j(t,\mathbf{x})\right) \end{array} \right) \end{array} \tag{7}$$

Because

$$\beta_{j,k}^{[s]*} = \beta_{k,j}^{[s]}$$

and $\varphi_j(t,\pm\infty) = 0$ then from (7):

$$\overline{\mathbf{v}}(t) = -\int d^3\mathbf{x} \cdot \sum_{\alpha=1}^{3} \mathbf{e}_\alpha \sum_{k=1}^{4} \sum_{j=1}^{4} \varphi_j^* \beta_{j,k}^{[\alpha]} \varphi_k.$$

If denote:

$$j_\alpha \stackrel{Def}{=} -\sum_{k=1}^{4} \sum_{j=1}^{4} \varphi_j^* \beta_{j,k}^{[\alpha]} \varphi_k \tag{8}$$

then

$$\mathbf{j} = j_1\mathbf{e}_1 + j_2\mathbf{e}_2 + j_3\mathbf{e}_3$$

is *the probability current* for which:

$$\overline{\mathbf{v}}(t) = \int d^3\mathbf{x} \cdot \mathbf{j}$$

If

$$\mathbf{j} \overset{Def}{=} \rho\mathbf{u} \tag{9}$$

then

$$\overline{\mathbf{v}}(t) = \int d^3\mathbf{x} \cdot \rho\mathbf{u},$$

that is $\mathbf{u}$ is *an event* α_1 *local probability velocity.*
Let $j \in \{1,2,3,4\}$, $k \in \{1,2,3,4\}$
Let

$$\varphi_j(t,\mathbf{x}) = \sum_{w,\mathbf{p}} c_{j,w,\mathbf{p}}\varsigma_{w,\mathbf{p}}(t,\mathbf{x})$$

with $\varsigma_{w,\mathbf{p}}(t,\mathbf{x}) \overset{def}{=} \exp(\mathrm{i}h(wt - \mathbf{px}))$ be the Fourier series for $\varphi_j(t,\mathbf{x})$.
Let $\varphi_{j,w,\mathbf{p}}(t,\mathbf{x}) \overset{def}{=} c_{j,w,\mathbf{p}}\varsigma_{w,\mathbf{p}}(t,\mathbf{x})$.
Let $\langle t,\mathbf{x}\rangle$ be any space-time point.
Denote value of function $\varphi_{k,w,\mathbf{p}}$ at this point as

$$\varphi_{k,w,\mathbf{p}}|_{\langle t,\mathbf{x}\rangle} = A_k$$

and value of function $\partial_t\varphi_{j,w,\mathbf{p}} - \sum_{s=1}^4 \sum_{\alpha=1}^3 \beta_{j,s}^{[\alpha]}\partial_\alpha\varphi_{s,w,\mathbf{p}}$ at this point as

$$\left(\partial_t\varphi_{j,w,\mathbf{p}} - \sum_{s=1}^4 \sum_{\alpha=1}^3 \beta_{j,s}^{[\alpha]}\partial_\alpha\varphi_{s,w,\mathbf{p}}\right)|_{\langle t,\mathbf{x}\rangle} = C_j.$$

There A_k and C_j are complex numbers. Hence the following equations set:

$$\left\{\begin{array}{c} \sum_{k=1}^4 z_{j,k,w,\mathbf{p}}A_k = C_j, \\ z_{j,k,w,\mathbf{p}}^* = -z_{k,j,w,\mathbf{p}} \end{array}\right| \tag{10}$$

is a set of 20 algebraic complex equations with 16 complex unknown numbers $z_{k,j,w,\mathbf{p}}$. This set can be reformulated as the set of 8 linear real equations with 16 real unknown numbers $\mathrm{Re}\left(z_{j,k,w,\mathbf{p}}\right)$ for $j<k$ and $\mathrm{Im}\left(z_{j,k,w,\mathbf{p}}\right)$ for $j\leq k$. This set has got solutions by the Kronecker-Capelli theorem. Hence at every point $\langle t,\mathbf{x}\rangle$ such complex number $z_{j,k,w,\mathbf{p}}|_{\langle t,\mathbf{x}\rangle}$ exists.

Let $\kappa_{w,\mathbf{p}}$ be a linear operator on the linear space, spanned by functions $\varsigma_{w,\mathbf{p}}\left(t,\mathbf{x}\right)$, and

$$\kappa_{w,\mathbf{p}}\varsigma_{w',\mathbf{p}'}\overset{def}{=}\left\{\begin{array}{c}\varsigma_{w',\mathbf{p}'},\text{ if }w=w',\ \mathbf{p}=\mathbf{p}';\\ 0,\text{ if }w\neq w'\text{or/and }\mathbf{p}\neq\mathbf{p}'\end{array}\right..$$

Let $Q_{j,k}$ be a operator such that in every point $\langle t,\mathbf{x}\rangle$:

$$Q_{j,k}|_{\langle t,\mathbf{x}\rangle}\overset{def}{=}\sum_{w,\mathbf{p}}\left(z_{j,k,w,\mathbf{p}}|_{\langle t,\mathbf{x}\rangle}\right)\kappa_{w,\mathbf{p}}$$

Therefore for every φ there exist operator $Q_{j,k}$ such that the φ dependence upon t is characterized by the following differential equations[1]:

$$\partial_t\varphi_j=\sum_{k=1}^{4}\left(\beta_{j,k}^{[1]}\partial_1+\beta_{j,k}^{[2]}\partial_2+\beta_{j,k}^{[3]}\partial_3+Q_{j,k}\right)\varphi_k. \tag{11}$$

and $Q_{j,k}^{*}=\sum_{w,\mathbf{p}}\left(z_{j,k,w,\mathbf{p}}^{*}|_{\langle t,\mathbf{x}\rangle}\right)\kappa_{w,\mathbf{p}}=-Q_{k,j}$.

In that case if

$$\widehat{H}_{j,k}\overset{def}{=}\mathrm{i}\left(\beta_{j,k}^{[1]}\partial_1+\beta_{j,k}^{[2]}\partial_2+\beta_{j,k}^{[3]}\partial_3+Q_{j,k}\right)$$

then $\widehat{H}$ is called as *the hamiltonian* of the moving with equation (11).

If $\mathcal{H}\left(t,\mathbf{x}\right)$ is denoted as:

$$\mathcal{H}\left(t,\mathbf{x}\right)\overset{Def}{=}\sum_{j=1}^{4}\psi_j^{\dagger}\left(\mathbf{x}\right)\sum_{k=1}^{4}\widehat{H}_{j,k}\left(t,\mathbf{x}\right)\psi_k\left(\mathbf{x}\right) \tag{12}$$

then $\mathcal{H}\left(\tau,\mathbf{x}\right)$ is called as *the hamiltonian density.*

From (6):

$$-\mathrm{i}\int d^3\mathbf{x}\cdot\mathcal{H}\left(t,\mathbf{x}\right)\Psi\left(t,\mathbf{x}_0\right)=\partial_t\Psi\left(t,\mathbf{x}_0\right).$$

[1]This set of equations is similar to the Dirac equation with the mass matrix [11], [12], [13]. I choose form of this set of equations in order to describe behaviour of $\rho_{\wp}\left(t,\mathbf{x}\right)$ by spinors and by the Clifford's set elements.

Formula (11) has got the following matrix form:

$$\partial_t \varphi = \left(\beta^{[1]}\partial_1 + \beta^{[2]}\partial_2 + \beta^{[3]}\partial_3 + \widehat{Q}\right)\varphi, \tag{13}$$

with

$$\varphi = \left[\begin{array}{c} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{array}\right]$$

and

$$\widehat{Q} = \left[\begin{array}{cccc} \mathrm{i}\vartheta_{1,1} & \mathrm{i}\vartheta_{1,2} - \varpi_{1,2} & \mathrm{i}\vartheta_{1,3} - \varpi_{1,3} & \mathrm{i}\vartheta_{1,4} - \varpi_{1,4} \\ \mathrm{i}\vartheta_{1,2} + \varpi_{1,2} & \mathrm{i}\vartheta_{2,2} & \mathrm{i}\vartheta_{2,3} - \varpi_{2,3} & \mathrm{i}\vartheta_{2,4} - \varpi_{2,4} \\ \mathrm{i}\vartheta_{1,3} + \varpi_{1,3} & \mathrm{i}\vartheta_{2,3} + \varpi_{2,3} & \mathrm{i}\vartheta_{3,3} & \mathrm{i}\vartheta_{3,4} - \varpi_{3,4} \\ \mathrm{i}\vartheta_{1,4} + \varpi_{1,4} & \mathrm{i}\vartheta_{2,4} + \varpi_{2,4} & \mathrm{i}\vartheta_{3,4} + \varpi_{3,4} & \mathrm{i}\vartheta_{4,4} \end{array}\right]$$

with $\vartheta_{j,k} \overset{def}{=} \sum_{w,\mathbf{p}} \operatorname{Im}\left(z_{j,k,w,\mathbf{p}}\right)\kappa_{w,\mathbf{p}}$ and $\varpi_{j,k} \overset{def}{=} \sum_{w,\mathbf{p}} \operatorname{Re}\left(z_{j,k,w,\mathbf{p}}\right)\kappa_{w,\mathbf{p}}$.

Let Θ_0, Θ_3, Υ_0 and Υ_3 are the solution of the following system of equations:

$$\left\{\begin{array}{c} -\Theta_0 + \Theta_3 - \Upsilon_0 + \Upsilon_3 = \vartheta_{1,1}; \\ -\Theta_0 - \Theta_3 - \Upsilon_0 - \Upsilon_3 = \vartheta_{2,2}; \\ -\Theta_0 - \Theta_3 + \Upsilon_0 + \Upsilon_3 = \vartheta_{3,3}; \\ -\Theta_0 + \Theta_3 + \Upsilon_0 - \Upsilon_3 = \vartheta_{4,4}; \end{array}\right|$$

Θ_1 and Υ_1 are a solution of the following system of equations:

$$\left\{\begin{array}{c} \Theta_1 + \Upsilon_1 = \vartheta_{1,2}; \\ -\Theta_1 + \Upsilon_1 = \vartheta_{3,4}. \end{array}\right|$$

Θ_2 and Υ_2 are the solution of the following system of equations:

$$\left\{\begin{array}{c} -\Theta_2 - \Upsilon_2 = \varpi_{1,2}; \\ \Theta_2 - \Upsilon_2 = \varpi_{3,4}; \end{array}\right|$$

and further:

$$\left\{\begin{array}{c} M_0 + M_{3,0} = \vartheta_{1,3}; \\ M_0 - M_{3,0} = \vartheta_{2,4}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_4 + M_{3,4} = \varpi_{1,3}; \\ M_4 - M_{3,4} = \varpi_{2,4}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_{1,0} - M_{2,4} = \vartheta_{1,4}; \\ M_{1,0} + M_{2,4} = \vartheta_{2,3}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_{1,4} - M_{2,0} = \varpi_{1,4}; \\ M_{1,4} + M_{2,0} = \varpi_{2,3} \end{array}\right|$$

then from (13):

$$\begin{array}{c} \left(\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi = \\ = \left(\begin{array}{c} \sum_{k=1}^{3}\beta^{[2]}\left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right) + \mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} \\ -\mathrm{i}M_{1,0}\gamma_\zeta^{[0]} - \mathrm{i}M_{1,4}\zeta^{[4]} - \\ -\mathrm{i}M_{2,0}\gamma_\eta^{[0]} - \mathrm{i}M_{2,4}\eta^{[4]} - \\ -\mathrm{i}M_{3,0}\gamma_\theta^{[0]} - \mathrm{i}M_{3,4}\theta^{[4]} \end{array}\right)\varphi \end{array} \quad (14)$$

Here summands

$$\begin{array}{c} -\mathrm{i}M_{1,0}\gamma_\zeta^{[0]} - \mathrm{i}M_{1,4}\zeta^{[4]} - \\ -\mathrm{i}M_{2,0}\gamma_\eta^{[0]} - \mathrm{i}M_{2,4}\eta^{[4]} - \\ -\mathrm{i}M_{3,0}\gamma_\theta^{[0]} - \mathrm{i}M_{3,4}\theta^{[4]}. \end{array}$$

contain the chromatic pentads elements and

$$\sum_{k=1}^{3}\beta^{[2]}\left(\partial_k + \mathrm{i}\Theta_k + \mathrm{i}\Upsilon_k\gamma^{[5]}\right) + \mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]}$$

contain the light pentad elements, only. I denote the following sum

$$\widehat{H}_l \stackrel{Def}{=} \sum_{k=1}^{3}\beta^{[2]}\left(\mathrm{i}\partial_k - \Theta_k - \Upsilon_k\gamma^{[5]}\right) - M_0\gamma^{[0]} - M_4\beta^{[4]}$$

as *a leptonn hamiltonian.*

4.2 Rotation of x_5Ox_4 and B-bosonn

If denote (8):

$$j_4 \overset{Def}{=} -\varphi^{*}\beta^{[4]}\varphi \text{ and } j_5 \overset{Def}{=} -\varphi^{*}\gamma^{[0]}\varphi$$

and (9):

$$\rho u_4 \overset{Def}{=} j_4 \text{ and } \rho u_5 \overset{Def}{=} j_5,$$

then

$$u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2 = 1.$$

Hence of only all five elements of the Clifford pentad lends the entire kit of the velocity components and, for the completeness, yet two "space" coordinates x_5 and x_4 should be added to our three x_1, x_2, x_3.

Let us denote:

$$\widetilde{\varphi}\left(t, x_1, x_2, x_3, x_5, x_4\right) \overset{Def}{=} \varphi\left(t, x_1, x_2, x_3\right) \cdot$$
$$\cdot \left(\exp\left(-\mathrm{i}\left(x_5 M_0\left(t, x_1, x_2, x_3\right) + x_4 M_4\left(t, x_1, x_2, x_3\right)\right)\right)\right).$$

In this case from (14) for

$$\begin{gathered} M_{1,0} = 0,\ M_{1,4} = 0, \\ M_{2,0} = 0,\ M_{2,4} = 0, \\ M_{3,0} = 0,\ M_{3,4} = 0 \end{gathered}$$

the motion equation for the leptonn hamiltonian is the following:

$$\left(\sum_{\mu=0}^{3} \beta^{[\mu]}\left(\mathrm{i}\partial_\mu - \Theta_\mu - \Upsilon_\mu\gamma^{[5]}\right) + \gamma^{[0]}\mathrm{i}\partial_5 + \beta^{[4]}\mathrm{i}\partial_4\right)\widetilde{\varphi} = 0. \tag{15}$$

Let g_1 be a positive real number and for $\mu \in \{0,1,2,3\}$: F_μ and B_μ be the solutions of the following system of the equations:

$$\left\{\begin{array}{c} -0.5g_1 B_\mu + F_\mu = -\Theta_\mu - \Upsilon_\mu;; \\ -g_1 B_\mu + F_\mu = -\Theta_\mu + \Upsilon_\mu. \end{array}\right|$$

Let *a charge matrix* be denoted as the following:

$$Y \overset{Def}{=} -\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 2\cdot 1_2 \end{array}\right].$$

Hence from (15):

$$\left(\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1 Y B_\mu\right) + \gamma^{[0]}\mathrm{i}\partial_5 + \beta^{[4]}\mathrm{i}\partial_4\right)\widetilde{\varphi} = 0. \tag{16}$$

Let $\chi\left(t, x_1, x_2, x_3\right)$ be a real function and:

$$\widetilde{U}\left(\chi\right) \overset{Def}{=} \left[\begin{array}{cc} \exp\left(\mathrm{i}\frac{\chi}{2}\right)1_2 & 0_2 \\ 0_2 & \exp\left(\mathrm{i}\chi\right)1_2 \end{array}\right]. \tag{17}$$

Because

$$\partial_\mu \widetilde{U} = -\mathrm{i}\frac{\partial_\mu \chi}{2}Y\widetilde{U}$$

and

$$\begin{aligned}
\widetilde{U}^\dagger \gamma^{[0]}\widetilde{U} &= \gamma^{[0]}\cos\frac{\chi}{2} + \beta^{[4]}\sin\frac{\chi}{2}, \\
\widetilde{U}^\dagger \beta^{[4]}\widetilde{U} &= \beta^{[4]}\cos\frac{\chi}{2} - \gamma^{[0]}\sin\frac{\chi}{2}, \\
\widetilde{U}^\dagger\widetilde{U} &= 1_4, \\
\widetilde{U}^\dagger Y\widetilde{U} &= Y, \\
\beta^{[k]}\widetilde{U} &= \widetilde{U}\beta^{[k]}
\end{aligned}$$

for $k \in \{1,2,3\}$

then the motion equation (16) is invariant for the following transformation (rotation of $x_4 O x_5$):

$$\begin{aligned}
&x_4 \to x_4' = x_4\cos\frac{\chi}{2} - x_5\sin\frac{\chi}{2}; \\
&x_5 \to x_5' = x_5\cos\frac{\chi}{2} + x_4\sin\frac{\chi}{2}; \\
&x_\mu \to x_\mu' = x_\mu \text{ for } \mu \in \{0,1,2,3\}; \\
&Y \to Y' = \widetilde{U}^\dagger Y\widetilde{U} = Y;
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\widetilde{\varphi} \to \widetilde{\varphi}' = \widetilde{U}\widetilde{\varphi},\\
&B_\mu \to B'_\mu = B_\mu - \frac{1}{g_1}\partial_\mu \chi,\\
&F_\mu \to F'_\mu = F_\mu.
\end{aligned}$$

4.3 Masses

Let a hamiltonian be called as *a Planck hamiltonian* if there exists a tiny positive real number h and functions $N_\vartheta(t, x_1, x_2, x_3)$ and $N_\varpi(t, x_1, x_2, x_3)$ such that $N_\vartheta(t, x_1, x_2, x_3)$ and $N_\varpi(t, x_1, x_2, x_3)$ have got a range of values in the set of the integer numbers and:

$$M_0 = N_\vartheta h \text{ and } M_4 = N_\varpi h.$$

Let

$$-\frac{\pi}{h} \le x_5 \le \frac{\pi}{h}, -\frac{\pi}{h} \le x_4 \le \frac{\pi}{h},$$

$$\widetilde{\varphi}\left(t, x_1, x_2, x_3, \pm\frac{\pi}{h}, x_4\right) = 0 \text{ and } \widetilde{\varphi}\left(t, x_1, x_2, x_3, x_5, \pm\frac{\pi}{h}\right) = 0.$$

In that case the Fourier series for $\widetilde{\varphi}$ is of the following form:

$$\begin{gathered}
\widetilde{\varphi}(t, x_1, x_2, x_3, x_5, x_4) = \varphi(t, x_1, x_2, x_3) \cdot \\
\cdot \sum_{n,s} \delta_{-n,N_\vartheta(t,\mathbf{x})}\delta_{-s,N_\varpi(t,\mathbf{x})} \exp\left(-\mathrm{i}h\left(nx_5 + sx_4\right)\right)
\end{gathered}$$

with

$$\begin{aligned}
\delta_{-n,N_\vartheta} &= \frac{h}{2\pi}\int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} \exp\left(\mathrm{i}h\left(nx_5\right)\right)\exp\left(\mathrm{i}N_\vartheta h x_5\right) dx_5 = \frac{\sin\left(\pi\left(n + N_\vartheta\right)\right)}{\pi\left(n + N_\vartheta\right)},\\
\delta_{-s,N_\varpi} &= \frac{h}{2\pi}\int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} \exp\left(\mathrm{i}h\left(sx_4\right)\right)\exp\left(\mathrm{i}N_\varpi h x_4\right) dx_4 = \frac{\sin\left(\pi\left(s + N_\varpi\right)\right)}{\pi\left(s + N_\varpi\right)}.
\end{aligned}$$

If denote:

$$\phi(t, \mathbf{x}, -n, -s) \overset{Def}{=} \varphi(t, \mathbf{x})\, \delta_{n,N_\vartheta(t,\mathbf{x})}\delta_{s,N_\varpi(t,\mathbf{x})}$$

then

$$\begin{array}{c}\widetilde{\varphi}\left(t,\mathbf{x},x_5,x_4\right)=\\=\sum_{n,s}\phi\left(t,\mathbf{x},n,s\right)\exp\left(-\mathrm{i}h\left(nx_5+sx_4\right)\right).\end{array} \tag{19}$$

The integer numbers n and s be denoted as *the mass numbers.*

From the properties of δ: in every point $\langle t,\mathbf{x}\rangle$: either

$$\widetilde{\varphi}\left(t,\mathbf{x},x_5,x_4\right)=0$$

or an integer numbers n_0 and s_0 exist for which:

$$\begin{array}{c}\widetilde{\varphi}\left(t,\mathbf{x},x_5,x_4\right)=\\=\phi\left(t,\mathbf{x},n_0,s_0\right)\exp\left(-\mathrm{i}h\left(n_0x_5+s_0x_4\right)\right).\end{array} \tag{20}$$

Here if

$$m_0\stackrel{Def}{=}\sqrt{n_0^2+s_0^2}$$

and

$$m\stackrel{Def}{=}hm_0$$

then m is denoted as *a mass* of $\widetilde{\varphi}$.

That is for the every space-time point: either this point is empty or single mass is placed in this point.

The motion equation (16) under the transformation (18) has got the following form:

$$\begin{array}{c}\sum_{n',s'}\left(\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\mathrm{i}\partial_\mu+F_\mu+0.5g_1YB_\mu\gamma^{[5]}\right)+\gamma^{[0]}\mathrm{i}\partial_5'+\beta^{[4]}\mathrm{i}\partial_4'\right)\cdot\\\cdot\exp\left(-\mathrm{i}h\left(n'x_5+s'x_4\right)\right)\widetilde{U}\phi=0\end{array}$$

with:

$$\begin{array}{c}n'=n\cos\frac{\chi}{2}-s\sin\frac{\chi}{2},\\s'=n\sin\frac{\chi}{2}+s\cos\frac{\chi}{2}.\end{array}$$

But s and n are an integer numbers and s' and n' must be an integer numbers, too.

A triplet $\langle\lambda;n,s\rangle$ of integer numbers is the Pythagorean triple with father λ.

Denumerable many of father numbers with a precise ε exist for every ε.

Let on the space of spinors $\widetilde{\varphi}$ the scalar product $\widetilde{\varphi} * \widetilde{\chi}$ be denoted as the following:

$$\widetilde{\varphi} * \widetilde{\chi} \stackrel{Def}{=} \left(\frac{h}{2\pi}\right)^2 \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} dx_5 \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} dx_4 \cdot \left(\widetilde{\varphi}^{\dagger} \cdot \widetilde{\chi}\right).$$

In this case:

$$\widetilde{\varphi} * \beta^{[k]} \widetilde{\varphi} = \varphi^{\dagger} \beta^{[k]} \varphi.$$

for $k \in \{1, 2, 3\}$.

Hence from (8):

$$\begin{aligned}
&\widetilde{\varphi} * \widetilde{\varphi} = \rho, \\
&\widetilde{\varphi} * \beta^{[1]} \widetilde{\varphi} = -j_1, \\
&\widetilde{\varphi} * \beta^{[2]} \widetilde{\varphi} = -j_2, \\
&\widetilde{\varphi}^{\dagger} * \beta^{[3]} \widetilde{\varphi} = -j_3.
\end{aligned}$$

4.3.1 The one-mass state

Let ϵ_μ ($\mu \in \{1, 2, 3, 4\}$) be a basis such that in this basis the light pentad has got a form (1) and let (19):

$$\widetilde{\varphi}\left(t, \mathbf{x}, x_5, x_4\right) = \exp\left(-\mathrm{i} h n x_5\right) \sum_{j=1}^{4} \phi_j \left(t, \mathbf{x}, n, 0\right) \epsilon_j.$$

In that case the hamiltonian has got the following form (from 16):

$$\widehat{H} = \sum_{k=1}^{3} \beta^{[k]} \mathrm{i} \partial_k + h n \gamma^{[0]} + \widehat{G}$$

with

$$\widehat{G} \stackrel{Def}{=} \sum_{\mu=0}^{3} \beta^{[\mu]} \left(F_\mu + 0.5 g_1 Y B_\mu\right).$$

If

$$\widehat{H}_0 \overset{Def}{=} \sum_{k=1}^{3} \beta^{[k]} \mathrm{i}\partial_k + hn\gamma^{[0]} \tag{21}$$

then the functions

$$u_1(\mathbf{k}) \exp(-\mathrm{i}h\mathbf{kx}) \text{ and } u_2(\mathbf{k}) \exp(-\mathrm{i}h\mathbf{kx})$$

with

$$u_1(\mathbf{k}) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} \omega(\mathbf{k})+n+k_3 \\ k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix}$$

and

$$u_2(\mathbf{k}) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n+k_3 \end{bmatrix}$$

are the eigenvectors of $\widehat{H}_0$ with the eigenvalue $\omega(\mathbf{k}) \overset{Def}{=} \sqrt{\mathbf{k}^2+n^2}$, and the functions

$$u_3(\mathbf{k}) \exp(-\mathrm{i}h\mathbf{kx}) \text{ and } u_4(\mathbf{k}) \exp(-\mathrm{i}h\mathbf{kx})$$

with

$$u_3(\mathbf{k}) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} -\omega(\mathbf{k})-n+k_3 \\ k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n+k_3 \\ k_1+\mathrm{i}k_2 \end{bmatrix}$$

and

$$u_4(\mathbf{k}) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} k_1-\mathrm{i}k_2 \\ -\omega(\mathbf{k})-n-k_3 \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \end{bmatrix}$$

are the eigenvectors of $\widehat{H}_0$ with the eigenvalue $-\omega(\mathbf{k})$.

Here $u_{\mu}(\mathbf{k})$ form an orthonormal basis in the space, spanned on vectors ϵ_{μ}.

Let :

$$b_{r,\mathbf{k}} \stackrel{Def}{=} \left(\frac{h}{2\pi}\right)^3 \sum_{j'=1}^{4} \int_{(V)} d^3\mathbf{x'} \cdot e^{\mathrm{i}h\mathbf{k}\mathbf{x'}} u^*_{r,j'}(\mathbf{k})\, \psi_{j'}(\mathbf{x'})$$

In that case because

$$\sum_{r=1}^{4} u^*_{r,j}(\mathbf{k})\, u_{r,j'}(\mathbf{k}) = \delta_{j,j'}$$

then

$$\psi_j(\mathbf{x}) = \sum_{\mathbf{k}} e^{-\mathrm{i}h\mathbf{k}\mathbf{x}} \sum_{r=1}^{4} b_{r,\mathbf{k}} u_{r,j}(\mathbf{k}) \tag{22}$$

and

$$\begin{aligned}
&\left\{b^{\dagger}_{s,\mathbf{k'}}, b_{r,\mathbf{k}}\right\} = \left(\frac{h}{2\pi}\right)^3 \delta_{s,r}\delta_{\mathbf{k},\mathbf{k'}}, \\
&\left\{b^{\dagger}_{s,\mathbf{k'}}, b^{\dagger}_{r,\mathbf{k}}\right\} = 0 = \left\{b_{s,\mathbf{k'}}, b_{r,\mathbf{k}}\right\}, \\
&b_{r,\mathbf{k}}\Phi_0 = 0.
\end{aligned} \tag{23}$$

The hamiltonian density (12) for $\widehat{H}_0$ is the following:

$$\mathcal{H}_0(\mathbf{x}) = \sum_{j=1}^{4} \psi^{\dagger}_j(\mathbf{x}) \sum_{k=1}^{4} \widehat{H}_{0,j,k}\psi_k(\mathbf{x}).$$

Hence from (22):

$$\int_{(V)} d^3\mathbf{x} \cdot \mathcal{H}_0(\mathbf{x}) = \left(\frac{2\pi}{h}\right)^3 \sum_{\mathbf{k}} h\omega(\mathbf{k}) \cdot \left(\sum_{r=1}^{2} b^{\dagger}_{r,\mathbf{k}} b_{r,\mathbf{k}} - \sum_{r=3}^{4} b^{\dagger}_{r,\mathbf{k}} b_{r,\mathbf{k}}\right)$$

Let a Fourier transformation for φ be the following:

$$\varphi_j(t,\mathbf{x}) = \sum_{\mathbf{p}} \sum_{r=1}^{4} c_r(t,\mathbf{p})\, u_{r,j}(\mathbf{p})\, e^{-\mathrm{i}h\mathbf{p}\mathbf{x}}$$

with

$$c_r(t,\mathbf{p}) \overset{Def}{=} \left(\frac{h}{2\pi}\right)^3 \sum_{j'=1}^{4} \int_{(V)} d^3\mathbf{x}' \cdot u^*_{r,j'}(\mathbf{p})\,\varphi_{j'}(t,\mathbf{x}')\,e^{\mathrm{i}h\mathbf{p}\mathbf{x}'}$$

I denote a function $\varphi_j(t,\mathbf{x})$ as *ordinary function* if there exists a real positive number L such that

if $|p_1| > L$ or/and $|p_2| > L$ or/and $|p_3| > L$ then $c_r(t,\mathbf{p}) = 0$.

In that case I denote:

$$\sum_{\mathbf{p}\in\mathbf{\Xi}} \overset{Def}{=} \sum_{p_1=-L}^{L} \sum_{p_2=-L}^{L} \sum_{p_3=-L}^{L}$$

If $\varphi_j(t,\mathbf{x})$ is an ordinary function then:

$$\varphi_j(t,\mathbf{x}) = \sum_{\mathbf{p}\in\mathbf{\Xi}} \sum_{r=1}^{4} c_r(t,\mathbf{p})\,u_{r,j}(\mathbf{p})\,e^{-\mathrm{i}h\mathbf{p}\mathbf{x}}.$$

Hence from (6):

$$\Psi(t,\mathbf{x}) = \sum_{\mathbf{p}} \sum_{r=1}^{4} \sum_{\mathbf{k}} \sum_{r'=1}^{4} c_r(t,\mathbf{p})\,e^{\mathrm{i}h(\mathbf{k}-\mathbf{p})\mathbf{x}} \sum_{j=1}^{4} u^*_{r',j}(\mathbf{k})\,u_{r,j}(\mathbf{p})\,b^\dagger_{r',\mathbf{k}}\Phi_0$$

and

$$\int_{(V)} d^3\mathbf{x} \cdot \Psi(t,\mathbf{x}) = \left(\frac{2\pi}{h}\right)^3 \sum_{\mathbf{p}} \sum_{r=1}^{4} c_r(t,\mathbf{p})\,b^\dagger_{r,\mathbf{p}}\Phi_0$$

If denote:

$$\widetilde{\Psi}(t,\mathbf{p}) \overset{Def}{=} \left(\frac{2\pi}{h}\right)^3 \sum_{r=1}^{4} c_r(t,\mathbf{p})\,b^\dagger_{r,\mathbf{p}}\Phi_0$$

then

$$\int_{(V)} d^3\mathbf{x} \cdot \Psi(t,\mathbf{x}) = \sum_{\mathbf{p}} \widetilde{\Psi}(t,\mathbf{p})$$

and

$$H_0\widetilde{\Psi}(t,\mathbf{p}) = \left(\frac{2\pi}{h}\right)^3 \sum_{\mathbf{k}} h\omega(\mathbf{k}) \cdot \left(\sum_{r=1}^{2} c_r(t,\mathbf{k})\,b^\dagger_{r,\mathbf{k}}\Phi_0 - \sum_{r=3}^{4} c_r(t,\mathbf{k})\,b^\dagger_{r,\mathbf{k}}\Phi_0\right)$$

H_0 is equivalent to operator:

$$\overset{\Xi}{H}_0 \overset{Def}{=} \left(\frac{2\pi}{h}\right)^3 \sum_{\mathbf{k}\in\Xi} h\omega\left(\mathbf{k}\right) \cdot \left(\sum_{r=1}^{2} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} - \sum_{r=3}^{4} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}}\right).$$

on set of ordinary functions.
Because (from (23))

$$b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} = \left(\frac{h}{2\pi}\right)^3 - b_{r,\mathbf{k}} b_{r,\mathbf{k}}^{\dagger}$$

then

$$\overset{\Xi}{H}_0 = \left(\frac{2\pi}{h}\right)^3 \sum_{\mathbf{k}\in\Xi} h\omega\left(\mathbf{k}\right) \left(\sum_{r=1}^{2} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} + \sum_{r=3}^{4} b_{r,\mathbf{k}} b_{r,\mathbf{k}}^{\dagger}\right) - h\sum_{\mathbf{k}\in\Xi} \omega\left(\mathbf{k}\right). \tag{24}$$

Let:

$$\begin{aligned} v_{(1)}\left(\mathbf{k}\right) &\overset{Def}{=} \gamma^{[0]} u_3\left(\mathbf{k}\right), \\ v_{(2)}\left(\mathbf{k}\right) &\overset{Def}{=} \gamma^{[0]} u_4\left(\mathbf{k}\right), \\ u_{(1)}\left(\mathbf{k}\right) &\overset{Def}{=} u_1\left(\mathbf{k}\right), \\ u_{(2)}\left(\mathbf{k}\right) &\overset{Def}{=} u_2\left(\mathbf{k}\right) \end{aligned} \tag{25}$$

and let:

$$\begin{aligned} d_1\left(\mathbf{k}\right) &\overset{Def}{=} -b_3^{\dagger}\left(-\mathbf{k}\right), \\ d_2\left(\mathbf{k}\right) &\overset{Def}{=} -b_4^{\dagger}\left(-\mathbf{k}\right). \end{aligned}$$

In that case:

$$\psi_j\left(\mathbf{x}\right) = \sum_{\mathbf{k}} \sum_{\alpha=1}^{2} \left(e^{-\mathrm{i}h\mathbf{kx}} b_{\alpha,\mathbf{k}} u_{(\alpha),j}\left(\mathbf{k}\right) + e^{\mathrm{i}h\mathbf{kx}} d_{\alpha,\mathbf{k}}^{\dagger} v_{(\alpha),j}\left(\mathbf{k}\right)\right)$$

and from (24) the Wick-ordering hamiltonian has got the following form:

$$:\overset{\Xi}{H}_0: = \left(\frac{2\pi}{h}\right)^3 h \sum_{\mathbf{k}\in\Xi} \omega\left(\mathbf{k}\right) \sum_{\alpha=1}^{2} \left(b_{\alpha,\mathbf{k}}^{\dagger} b_{\alpha,\mathbf{k}} + d_{\alpha,\mathbf{k}}^{\dagger} d_{\alpha,\mathbf{k}}\right).$$

Here $b^{\dagger}_{\alpha,\mathbf{k}}$ are *creation operators*, and $b_{\alpha,\mathbf{k}}$ are *annihilation operators* of *n-leptonn* with *a momentum* $\mathbf{k}$ and *a spin index* α; $d^{\dagger}_{\alpha,\mathbf{k}}$ are *creation operators*, and $d_{\alpha,\mathbf{k}}$ are *annihilation operators* of *anti-n-leptonn* with *a momentum* $\mathbf{k}$ and *a spin index* α.

Functions:

$$u_{(1)}(\mathbf{k})\exp(-\mathrm{i}h\mathbf{kx}) \text{ and } u_{(2)}(\mathbf{k})\exp(-\mathrm{i}h\mathbf{kx})$$

are *the basic n-leptonn functions* with momentum $\mathbf{k}$, and

$$v_{(1)}(\mathbf{k})\exp(\mathrm{i}h\mathbf{kx}) \text{ and } v_{(2)}(\mathbf{k})\exp(\mathrm{i}h\mathbf{kx})$$

are *the anti-n-leptonn basic functions* with momentum $\mathbf{k}$.

4.3.2 The bi-mass state

Let

$$\begin{aligned}
&\widetilde{\varphi}(t,\mathbf{x},x_5,x_4) = \\
&= \exp(-\mathrm{i}hsx_4)\sum_{r=1}^{4}\phi_r(t,\mathbf{x},0,s)\epsilon_r \\
&+ \exp(-\mathrm{i}hnx_5)\sum_{j=1}^{4}\phi_j(t,\mathbf{x},n,0)\epsilon_j \ .
\end{aligned} \tag{26}$$

Hence in the basis

$$\begin{aligned}
&\langle \exp(-\mathrm{i}hsx_4)\epsilon_1, \exp(-\mathrm{i}hsx_4)\epsilon_2, \exp(-\mathrm{i}hsx_4)\epsilon_3, \exp(-\mathrm{i}hsx_4)\epsilon_4, \\
&\exp(-\mathrm{i}hnx_5)\epsilon_1, \exp(-\mathrm{i}hnx_5)\epsilon_2, \exp(-\mathrm{i}hnx_5)\epsilon_3, \exp(-\mathrm{i}hnx_5)\epsilon_4\rangle
\end{aligned}$$

a 8-components bi-spinor:

$$\xi = \begin{bmatrix} \phi_1(0,s) \\ \phi_2(0,s) \\ \phi_3(0,s) \\ \phi_4(0,s) \\ \phi_1(n,0) \\ \phi_2(n,0) \\ \phi_3(n,0) \\ \phi_4(n,0) \end{bmatrix} \tag{27}$$

corresponds to $\widetilde{\varphi}$.
From (20): in every point $\langle t, \mathbf{x} \rangle$:

$$\text{either } \xi = \begin{bmatrix} \phi_1(0,s) \\ \phi_2(0,s) \\ \phi_3(0,s) \\ \phi_4(0,s) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ or } \xi = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \phi_1(n,0) \\ \phi_2(n,0) \\ \phi_3(n,0) \\ \phi_4(n,0) \end{bmatrix} \tag{28}$$

of δ characteristics.
Let us denote:

$$\phi_L \overset{Def}{=} \phi_1\epsilon_1 + \phi_2\epsilon_2 \text{ and } \phi_R \overset{Def}{=} \phi_3\epsilon_3 + \phi_4\epsilon_4.$$

Hence from (26):

$$\begin{aligned} \widetilde{\varphi}(x_5, x_4) = \exp(-\mathrm{i}hsx_4)(\phi_L(0,s) + \phi_R(0,s)) + \\ + \exp(-\mathrm{i}hnx_5)(\phi_L(n,0) + \phi_R(n,0)) \end{aligned} \tag{29}$$

and ξ can be denoted as the following:

$$\xi = \begin{pmatrix} \phi_L(0,s) \\ \phi_R(0,s) \\ \phi_L(n,0) \\ \phi_R(n,0) \end{pmatrix}. \tag{30}$$

If use denotation:

$$\underline{\varsigma} \overset{Def}{=} \begin{bmatrix} \varsigma & 0_4 \\ 0_4 & \varsigma \end{bmatrix},$$

then from (29) :

$$\begin{aligned} \xi^\dagger\xi &= \rho, \\ -\xi^\dagger\underline{\beta^{[1]}}\xi &= j_1, \\ -\xi^\dagger\underline{\beta^{[2]}}\xi &= j_2, \\ -\xi^\dagger\underline{\beta^{[3]}}\xi &= j_3. \end{aligned}$$

If U is an 8×8 complex matrix, $\xi' \overset{Def}{=} U\xi$ and

$$\begin{array}{r}\xi'^{\dagger}\xi' = \rho, \\ -\xi'^{\dagger}\underline{\beta^{[1]}}\xi' = j_1, \\ -\xi'^{\dagger}\underline{\beta^{[2]}}\xi' = j_2, \\ -\xi'^{\dagger}\underline{\beta^{[3]}}\xi' = j_3\end{array} \tag{31}$$

then for $k \in \{1, 2, 3\}$:

$$U^{\dagger}\underline{\beta^{[k]}}U = \underline{\beta^{[k]}}$$

and

$$U^{\dagger}U = 1_8.$$

There exist real functions $\chi(t, \mathbf{x})$, $\alpha(t, \mathbf{x})$, $a(t, \mathbf{x})$, $b(t, \mathbf{x})$, $c(t, \mathbf{x})$, $q(t, \mathbf{x})$, $u(t, \mathbf{x})$, $v(t, \mathbf{x})$, $k(t, \mathbf{x})$, $s(t, \mathbf{x})$ such that

$$U \overset{Def}{=} \underline{\widetilde{U}}U^{(\alpha)}U^{(-)}U^{(+)}$$

with

$$U^{(\alpha)} \overset{Def}{=} \exp(\mathrm{i}\alpha) 1_8,$$

$$U^{(-)} \overset{Def}{=} \begin{bmatrix} (a+\mathrm{i}b) 1_2 & 0_2 & (c+\mathrm{i}q) 1_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ (-c+\mathrm{i}q) 1_2 & 0_2 & (a-\mathrm{i}b) 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{bmatrix}$$

with

$$a^2 + b^2 + c^2 + q^2 = 1$$

and

$$U^{(+)} \overset{Def}{=} \begin{bmatrix} 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & (u+\mathrm{i}v) 1_2 & 0_2 & (k+\mathrm{i}s) 1_2 \\ 0_2 & 0_2 & 1_2 & 0_2 \\ 0_2 & (-k+\mathrm{i}s) 1_2 & 0_2 & (u-\mathrm{i}v) 1_2 \end{bmatrix} \tag{32}$$

with

$$u^2 + v^2 + k^2 + s^2 = 1.$$

$\widetilde{U}$ is denoted at (17)

4.3.3 The global $U^{(-)}$ transformation

The $U^{(-)}$ eigenvalues are:

$$w_\circ = a + \mathrm{i}\sqrt{(1-a^2)} \text{ and } w_* = a - \mathrm{i}\sqrt{(1-a^2)}.$$

Let us denote:

$$\ell_\circ \overset{Def}{=} \frac{1}{2\sqrt{(1-a^2)}} \left[\begin{array}{cc} \left(b + \sqrt{(1-a^2)}\right) 1_4 & (q - \mathrm{i}c)\, 1_4 \\ (q + \mathrm{i}c)\, 1_4 & \left(\sqrt{(1-a^2)} - b\right) 1_4 \end{array} \right],$$

$$\ell_* \overset{Def}{=} \frac{1}{2\sqrt{(1-a^2)}} \left[\begin{array}{cc} \left(\sqrt{(1-a^2)} - b\right) 1_4 & (-q + \mathrm{i}c)\, 1_4 \\ (-q - \mathrm{i}c)\, 1_4 & \left(b + \sqrt{(1-a^2)}\right) 1_4 \end{array} \right].$$

These operators are fulfilled to the following conditions:

$$\begin{aligned} \ell_\circ \ell_\circ = \ell_\circ,\ \ell_* \ell_* &= \ell_*; \\ \ell_\circ \ell_* = 0 &= \ell_* \ell_\circ, \\ (\ell_\circ - \ell_*)(\ell_\circ - \ell_*) &= 1_8, \\ \ell_\circ + \ell_* &= 1_8, \end{aligned}$$

$$\begin{aligned} \ell_\circ \underline{\gamma^{[0]}} &= \underline{\gamma^{[0]}} \ell_\circ,\ \ell_* \underline{\gamma^{[0]}} = \underline{\gamma^{[0]}} \ell_*, \\ \ell_\circ \underline{\beta^{[4]}} &= \underline{\beta^{[4]}} \ell_\circ,\ \ell_* \underline{\beta^{[4]}} = \underline{\beta^{[4]}} \ell_* \end{aligned}$$

and

$$\begin{aligned} U^{(-)\dagger} \underline{\gamma^{[0]}} U^{(-)} &= a \underline{\gamma^{[0]}} - (\ell_\circ - \ell_*) \sqrt{1-a^2} \underline{\beta^{[4]}}, \\ U^{(-)\dagger} \underline{\beta^{[4]}} U^{(-)} &= a \underline{\beta^{[4]}} + (\ell_\circ - \ell_*) \sqrt{1-a^2} \underline{\gamma^{[0]}}. \end{aligned} \tag{33}$$

From (16) the leptonn motion equation:

$$\left(\sum_{\mu=0}^{3} \beta^{[\mu]} \left(\mathrm{i}\partial_\mu + F_\mu + 0.5 g_1 Y B_\mu\right) + \left(\gamma^{[0]} \mathrm{i}\partial_5 + \beta^{[4]} \mathrm{i}\partial_4\right) \right) \widetilde{\varphi} = 0. \tag{34}$$

If

$$\partial_k U^{(-)\dagger} = U^{(-)\dagger}\partial_k$$

for $k \in \{0,1,2,3,4,5\}$ then

$$\left(\begin{array}{c} U^{(-)\dagger}\mathrm{i}\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1\underline{Y}B_\mu\right) \\ +\underline{\gamma^{[0]}}U^{(-)\dagger}\mathrm{i}\partial_5 + \underline{\beta^{[4]}}U^{(-)\dagger}\mathrm{i}\partial_4 \end{array}\right) U^{(-)}\widetilde{\varphi} = 0.$$

Hence from (33):

$$U^{(-)\dagger}\left(\begin{array}{c} \sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1\underline{Y}B_\mu\right) \\ +\underline{\gamma^{[0]}}\mathrm{i}\left(a\partial_5 - \left(\ell_\circ - \ell_*\right)\sqrt{1-a^2}\partial_4\right) \\ +\underline{\beta^{[4]}}\mathrm{i}\left(\sqrt{1-a^2}\left(\ell_\circ - \ell_*\right)\partial_5 + a\partial_4\right) \end{array}\right) U^{(-)}\widetilde{\varphi} = 0.$$

Thus, if to denote:

$$\begin{array}{c} x_4' = \left(\ell_\circ + \ell_*\right)ax_4 + \left(\ell_\circ - \ell_*\right)\sqrt{1-a^2}x_5, \\ x_5' = \left(\ell_\circ + \ell_*\right)ax_5 - \left(\ell_\circ - \ell_*\right)\sqrt{1-a^2}x_4 \end{array}$$

then

$$\left(\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1\underline{Y}B_\mu\right) + \left(\underline{\gamma^{[0]}}\mathrm{i}\partial_5' + \underline{\beta^{[4]}}\mathrm{i}\partial_4'\right)\right)\widetilde{\varphi}' = 0 \tag{35}$$

with

$$\widetilde{\varphi}' = U^{(-)}\widetilde{\varphi}.$$

That is the leptonn hamiltonian is invariant for the following global transformation:

$$\begin{array}{l} \widetilde{\varphi} \rightarrow \widetilde{\varphi}' = U^{(-)}\widetilde{\varphi}, \\ x_4 \rightarrow x_4' = \left(\ell_\circ + \ell_*\right)ax_4 + \left(\ell_\circ - \ell_*\right)\sqrt{1-a^2}x_5, \\ x_5 \rightarrow x_5' = \left(\ell_\circ + \ell_*\right)ax_5 - \left(\ell_\circ - \ell_*\right)\sqrt{1-a^2}x_4, \\ x_\mu \rightarrow x_\mu' = x_\mu. \end{array} \tag{36}$$

4.3.4 Neutrinno

From (29) for $s = n$:

$$\begin{aligned}\widetilde{\varphi} = &\exp\left(-\mathrm{i}hnx_4\right)\left(\phi_L\left(0,n\right)+\phi_R\left(0,n\right)\right)+\\&+\exp\left(-\mathrm{i}hnx_5\right)\left(\phi_L\left(n,0\right)+\phi_R\left(n,0\right)\right).\end{aligned}$$

If from (28): $\phi_L\left(0,n\right)=0$ and $\phi_R\left(0,n\right)=0$ then from (30):

$$\xi = \begin{pmatrix} 0 \\ 0 \\ \phi_L\left(n,0\right) \\ \phi_R\left(n,0\right) \end{pmatrix}.$$

Let

$$\underline{\widehat{H}}_{0,4} \overset{Def.}{=} \sum_{k=1}^{3} \underline{\beta^{[k]}}\mathrm{i}\partial_k + hn\left(\underline{\gamma^{[0]}} + \underline{\beta^{[4]}}\right).$$

The 8-vectors

$$\underline{u}_1\left(\mathbf{k}\right) \overset{Def}{=} \frac{1}{2\sqrt{\omega\left(\mathbf{k}\right)\left(\omega\left(\mathbf{k}\right)+n\right)}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega\left(\mathbf{k}\right)+n+k_3 \\ k_1+\mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right)+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix}$$

and

$$\underline{u}_2\left(\mathbf{k}\right) \overset{Def}{=} \frac{1}{2\sqrt{\omega\left(\mathbf{k}\right)\left(\omega\left(\mathbf{k}\right)+n\right)}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_1-\mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right)+n-k_3 \\ -k_1+\mathrm{i}k_2 \\ \omega\left(\mathbf{k}\right)+n+k_3 \end{bmatrix}$$

correspond to the eigenvectors of $\widehat{\underline{H}}_{0,4}$ with eigenvalue $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + n^2}$, and the 8-vectors

$$\underline{u}_3(\mathbf{k}) \stackrel{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\omega(\mathbf{k}) - n + k_3 \\ k_1 + \mathrm{i}k_2 \\ \omega(\mathbf{k}) + n + k_3 \\ k_1 + \mathrm{i}k_2 \end{bmatrix}$$

and

$$\underline{u}_4(\mathbf{k}) \stackrel{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_1 - \mathrm{i}k_2 \\ -\omega(\mathbf{k}) - n - k_3 \\ k_1 - \mathrm{i}k_2 \\ \omega(\mathbf{k}) + n - k_3 \end{bmatrix}$$

correspond to the eigenvectors of $\widehat{\underline{H}}_{0,4}$ with eigenvalue $-\omega(\mathbf{k})$.
Let

$$\begin{aligned} \widehat{\underline{H}}'_{0,4} &\stackrel{Def}{=} U^{(-)}\widehat{\underline{H}}_{0,4}U^{(-)\dagger}, \\ \underline{u}'_\mu(\mathbf{k}) &\stackrel{Def}{=} U^{(-)}\underline{u}_\mu(\mathbf{k}). \end{aligned}$$

That is

$$\underline{u}'_1(\mathbf{k}) = \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} (c+\mathrm{i}q)(\omega(\mathbf{k}) + n + k_3) \\ (c+\mathrm{i}q)(k_1 + \mathrm{i}k_2) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(\omega(\mathbf{k}) + n + k_3) \\ (a-\mathrm{i}b)(k_1 + \mathrm{i}k_2) \\ \omega(\mathbf{k}) + n - k_3 \\ -k_1 - \mathrm{i}k_2 \end{bmatrix},$$

$$\underline{u}'_2(\mathbf{k}) = \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ (c+\mathrm{i}q)(\omega(\mathbf{k})+n-k_3) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ (a-\mathrm{i}b)(\omega(\mathbf{k})+n-k_3) \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n+k_3 \end{bmatrix},$$

$$\underline{u}'_3(\mathbf{k}) = \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} -(c+\mathrm{i}q)(\omega(\mathbf{k})+n-k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ 0 \\ 0 \\ -(a-\mathrm{i}b)(\omega(\mathbf{k})+n-k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k})+n+k_3 \\ k_1+\mathrm{i}k_2 \end{bmatrix},$$

$$\underline{u}'_4(\mathbf{k}) = \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ -(c+\mathrm{i}q)(\omega(\mathbf{k})+n+k_3) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ -(a-\mathrm{i}b)(\omega(\mathbf{k})+n+k_3) \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \end{bmatrix}.$$

Here $\underline{u}'_1(\mathbf{k})$ and $\underline{u}'_2(\mathbf{k})$ correspond to the eigenvectors of $\underline{\widehat{H}}'_{0,4}$ with eigenvalue $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2+n^2}$, and $\underline{u}'_3(\mathbf{k})$ and $\underline{u}'_4(\mathbf{k})$ correspond to the eigenvectors of $\underline{\widehat{H}}'_{0,4}$ with eigenvalue $-\omega(\mathbf{k})$.

Let as in (25):

$$\underline{v}_{(1)}(\mathbf{k}) \overset{Def}{=} \underline{\gamma^{[0]}}\underline{u}'_3(\mathbf{k}),$$
$$\underline{v}_{(2)}(\mathbf{k}) \overset{Def}{=} \underline{\gamma^{[0]}}\underline{u}'_4(\mathbf{k}),$$

$$\underline{u}_{(1)}(\mathbf{k}) \overset{Def}{=} \underline{u}'_1(\mathbf{k}),$$
$$\underline{u}_{(2)}(\mathbf{k}) \overset{Def}{=} \underline{u}'_2(\mathbf{k}).$$

Hence

$$\underline{v}_{(1)}(\mathbf{k}) = \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} 0 \\ 0 \\ -(c+\mathrm{i}q)(\omega(\mathbf{k})+n-k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k})+n+k_3 \\ k_1+\mathrm{i}k_2 \\ -(a-\mathrm{i}b)(\omega(\mathbf{k})+n-k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \end{bmatrix}$$

and

$$\underline{v}_{(2)}(\mathbf{k}) = \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} 0 \\ 0 \\ (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ -(c+\mathrm{i}q)(\omega(\mathbf{k})+n+k_3) \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ -(a-\mathrm{i}b)(\omega(\mathbf{k})+n+k_3) \end{bmatrix}.$$

$\underline{u}'_{(\alpha)}(\mathbf{k})$ are denoted as *bi-n-leptonn* and $\underline{v}_{(\alpha)}(\mathbf{k})$ - as *bi-anti-n-leptonn* basic vectors with momentum $\mathbf{k}$ and spin index α.

Hence bi-anti-n-leptonn basic vectors are the result of the acting of $U^{(+)}$ (32).

The vectors

$$\begin{aligned} l_{n,(1)}(\mathbf{k}) &= \begin{bmatrix} (a-\mathrm{i}b)(\omega(\mathbf{k})+n+k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k})+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix} \text{ and} \\ l_{n,(2)}(\mathbf{k}) &= \begin{bmatrix} (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ (a-\mathrm{i}b)(\omega(\mathbf{k})+n-k_3) \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n+k_3 \end{bmatrix} \end{aligned}$$

are denoted as *leptonn components* of bi-n-leptonn basic vectors, and vectors

$$\nu_{n,(1)}(\mathbf{k}) = \begin{bmatrix} \omega(\mathbf{k}) + n + k_3 \\ k_1 + \mathrm{i}k_2 \\ 0 \\ 0 \end{bmatrix} \text{ and } \nu_{n,(2)}(\mathbf{k}) = \begin{bmatrix} k_1 - \mathrm{i}k_2 \\ \omega(\mathbf{k}) + n - k_3 \\ 0 \\ 0 \end{bmatrix}$$

are denoted as *neutrinno components* of bi-n-leptonn basic vectors.

The vectors

$$\begin{aligned} \bar{l}_{n,(1)}(\mathbf{k}) &= \begin{bmatrix} \omega(\mathbf{k}) + n + k_3 \\ k_1 + \mathrm{i}k_2 \\ -(a - \mathrm{i}b)(\omega(\mathbf{k}) + n - k_3) \\ (a - \mathrm{i}b)(k_1 + \mathrm{i}k_2) \end{bmatrix} \text{ and} \\ \bar{l}_{n,(2)}(\mathbf{k}) &= \begin{bmatrix} k_1 - \mathrm{i}k_2 \\ \omega(\mathbf{k}) + n - k_3 \\ (a - \mathrm{i}b)(k_1 - \mathrm{i}k_2) \\ -(a - \mathrm{i}b)(\omega(\mathbf{k}) + n + k_3) \end{bmatrix} \end{aligned}$$

are denoted as *leptonn components* of anti-bi-n-leptonn basic vectors, and vectors

$$\overline{\nu}_{n,(1)}(\mathbf{k}) = \begin{bmatrix} 0 \\ 0 \\ -(\omega(\mathbf{k}) + n - k_3) \\ k_1 + \mathrm{i}k_2 \end{bmatrix}$$

and

$$\overline{\nu}_{n,(2)}(\mathbf{k}) = \begin{bmatrix} 0 \\ 0 \\ k_1 - \mathrm{i}k_2 \\ -(\omega(\mathbf{k}) + n + k_3) \end{bmatrix}$$

are denoted as *neutrinno components* of anti-bi-n-leptonn basic vectors.

4.3.5 Local $U^{(-)}$, W, Z and A-bozons

From (34):

$$\left(\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\left(\mathrm{i}\partial_\mu+F_\mu+0.5g_1\underline{Y}B_\mu\right)+\underline{\gamma^{[0]}}\mathrm{i}\partial_5+\underline{\beta^{[4]}}\mathrm{i}\partial_4\right)\widetilde{\varphi}=0.$$

Let:

$$K\overset{Def}{=}\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\left(F_\mu+0.5g_1\underline{Y}B_\mu\right). \tag{37}$$

In that case the motion equation has got the following form:

$$\left(K+\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\mathrm{i}\partial_\mu+\underline{\gamma^{[0]}}\mathrm{i}\partial_5+\underline{\beta^{[4]}}\mathrm{i}\partial_4\right)\widetilde{\varphi}=0. \tag{38}$$

Hence for the following transformations:

$$\begin{aligned}
&\widetilde{\varphi}\to\widetilde{\varphi}'\overset{Def}{=}U^{(-)}\widetilde{\varphi},\\
&x_4\to x_4'\overset{Def}{=}\left(\ell_\circ+\ell_*\right)ax_4+\left(\ell_\circ-\ell_*\right)\sqrt{1-a^2}x_5,\\
&x_5\to x_5'\overset{Def}{=}\left(\ell_\circ+\ell_*\right)ax_5-\left(\ell_\circ-\ell_*\right)\sqrt{1-a^2}x_4,\\
&x_\mu\to x_\mu'\overset{Def}{=}x_\mu,\text{ for }\mu\in\{0,1,2,3\},\\
&K\to K'
\end{aligned} \tag{39}$$

with

$$\partial_4U^{(-)}=U^{(-)}\partial_4\text{ and }\partial_5U^{(-)}=U^{(-)}\partial_5$$

this equation has got the following form:

$$\left(\begin{array}{c}U^{(-)\dagger}K'U^{(-)}+\\+\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\mathrm{i}\left(\partial_\mu+U^{(-)\dagger}\left(\partial_\mu U^{(-)}\right)\right)+\underline{\gamma^{[0]}}\mathrm{i}\partial_5+\underline{\beta^{[4]}}\mathrm{i}\partial_4\end{array}\right)\widetilde{\varphi}=0. \tag{40}$$

Therefore if

$$K'=K-\mathrm{i}\sum_{\mu=0}^{3}\underline{\beta^{[\mu]}}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger} \tag{41}$$

then the equation (38) is invariant for the local transformation (39).
Let g_2 be a positive real number.
If design (a, b, c, q form $U^{(-)}$):

$$\begin{array}{l} W_{\mu}^{0,} \overset{Def}{=} -2\frac{1}{g_2 q} \left(\begin{array}{c} q\left(\partial_{\mu}a\right)b - q\left(\partial_{\mu}b\right)a + \left(\partial_{\mu}c\right)q^2 + \\ +a\left(\partial_{\mu}a\right)c + b\left(\partial_{\mu}b\right)c + c^2\left(\partial_{\mu}c\right) \end{array} \right) \\ W_{\mu}^{1,} \overset{Def}{=} -2\frac{1}{g_2 q} \left(\begin{array}{c} \left(\partial_{\mu}a\right)a^2 - bq\left(\partial_{\mu}c\right) + a\left(\partial_{\mu}b\right)b + \\ +a\left(\partial_{\mu}c\right)c + q^2\left(\partial_{\mu}a\right) + c\left(\partial_{\mu}b\right)q \end{array} \right) \\ W_{\mu}^{2,} \overset{Def}{=} -2\frac{1}{g_2 q} \left(\begin{array}{c} q\left(\partial_{\mu}a\right)c - a\left(\partial_{\mu}a\right)b - b^2\left(\partial_{\mu}b\right) - \\ -c\left(\partial_{\mu}c\right)b - \left(\partial_{\mu}b\right)q^2 - \left(\partial_{\mu}c\right)qa \end{array} \right) \end{array}$$

and

$$W_{\mu} \overset{Def}{=} \left[\begin{array}{cccc} W_{\mu}^{0,}1_2 & 0_2 & \left(W_{\mu}^{1,} - \mathrm{i}W_{\mu}^{2,}\right)1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \\ \left(W_{\mu}^{1,} + \mathrm{i}W_{\mu}^{2,}\right)1_2 & 0_2 & -W_{\mu}^{0,}1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \end{array} \right]$$

then

$$-\mathrm{i}\left(\partial_{\mu}U^{(-)}\right)U^{(-)\dagger} = \frac{1}{2}g_2 W_{\mu}, \tag{42}$$

and from (42), (37), (41), (38):

$$\left(\begin{array}{c} \sum_{\mu=0}^{3} \underline{\beta^{[\mu]}}\mathrm{i}\left(\partial_{\mu} - \mathrm{i}0.5g_1 B_{\mu}\underline{Y} - \mathrm{i}\frac{1}{2}g_2 W_{\mu} - \mathrm{i}F_{\mu}\right) \\ +\underline{\gamma^{[0]}}\mathrm{i}\partial_5' + \underline{\beta^{[4]}}\mathrm{i}\partial_4' \end{array} \right)\widetilde{\varphi}' = 0. \tag{43}$$

Let

$$U' \overset{Def}{=} \left[\begin{array}{cccc} \left(a' + \mathrm{i}b'\right)1_2 & 0_2 & \left(c' + \mathrm{i}q'\right)1_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ \left(-c' + \mathrm{i}q'\right)1_2 & 0_2 & \left(a' - \mathrm{i}b'\right)1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{array} \right].$$

In this case if

$$U'' \overset{Def}{=} U'U^{(-)}$$

then U'' has got the similar form:

$$U'' \stackrel{Def}{=} \left[\begin{array}{cccc} (a''+\mathrm{i}b'')\,1_2 & 0_2 & (c''+\mathrm{i}q'')\,1_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ (-c''+\mathrm{i}q'')\,1_2 & 0_2 & (a''-\mathrm{i}b'')\,1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{array} \right].$$

If

$$\ell''_\circ \stackrel{Def}{=} \frac{1}{2\sqrt{(1-a''^2)}} \left[\begin{array}{cc} \left(b''+\sqrt{(1-a''^2)}\right) 1_4 & (q''-\mathrm{i}c'')\,1_4 \\ (q''+\mathrm{i}c'')\,1_4 & \left(\sqrt{(1-a''^2)}-b''\right) 1_4 \end{array} \right],$$

$$\ell''_* \stackrel{Def}{=} \frac{1}{2\sqrt{(1-a''^2)}} \left[\begin{array}{cc} \left(\sqrt{(1-a''^2)}-b''\right) 1_4 & (-q''+\mathrm{i}c'')\,1_4 \\ (-q''-\mathrm{i}c'')\,1_4 & \left(b''+\sqrt{(1-a''^2)}\right) 1_4 \end{array} \right];$$

$$\begin{aligned} &\widetilde{\varphi} \rightarrow \widetilde{\varphi}'' \stackrel{Def}{=} U''\widetilde{\varphi}, \\ &x_4 \rightarrow x''_4 \stackrel{Def}{=} (\ell''_\circ + \ell''_*)\, a'' x_4 + (\ell''_\circ - \ell''_*)\sqrt{1-a''^2} x_5, \\ &x_5 \rightarrow x''_5 \stackrel{Def}{=} (\ell''_\circ + \ell''_*)\, a'' x_5 - (\ell''_\circ - \ell''_*)\sqrt{1-a''^2} x_4, \\ &x_\mu \rightarrow x''_\mu \stackrel{Def}{=} x_\mu, \text{ for } \mu \in \{0,1,2,3\}, \\ &K \rightarrow K'' \stackrel{Def}{=} \sum_{\mu=0}^{3} \underline{\beta^{[\mu]}} \left(F_\mu + 0.5 g_1 \underline{Y} B_\mu + \frac{1}{2} g_2 W''_\mu\right) \end{aligned} \tag{44}$$

then from (42):

$$W''_\mu = -\frac{2i}{g_2}\left(\partial_\mu \left(U'U^{(-)}\right)\right)\left(U'U^{(-)}\right)^\dagger.$$

Hence:

$$W''_\mu = -\frac{2i}{g_2}\left(\partial_\mu U'\right) U'^\dagger - \frac{2i}{g_2} U' \left(\partial_\mu U^{(-)}\right) U^{(-)\dagger} U'^\dagger,$$

i.e. from (42):

$$W''_\mu = U' W_\mu U'^\dagger - \frac{2i}{g_2}\left(\partial_\mu U'\right) U'^\dagger. \tag{45}$$

If

$$W_{\mu,\nu} \stackrel{Def}{=} \left(\partial_\mu W_\nu - \partial_\nu W_\mu - i\frac{g_2}{2}\left(W_\mu W_\nu - W_\nu W_\mu\right)\right)$$

then

$$W''_{\mu,\nu} = \partial_\mu W''_\nu - \partial_\nu W''_\mu - i\frac{g_2}{2}\left(W''_\mu W''_\nu - W''_\nu W''_\mu\right) = U' W'_{\mu,\nu} U'^{\dagger}.$$

Therefore $W_{\mu,\nu}$ is invariant for the transformation (44).

The motion equation of the Yang-Mills SU(2) field in the space without matter (for instance [15] or [14]) has got the following form:

$$\partial^\nu \mathbf{W}_{\mu\nu} = -g_2 \mathbf{W}^\nu \times \mathbf{W}_{\mu\nu}$$

with:

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + g_2 \mathbf{W}_\mu \times \mathbf{W}_\nu$$

and

$$\mathbf{W}_\mu = \begin{bmatrix} W^{0,}_\mu \\ W^{1,}_\mu \\ W^{2,}_\mu \end{bmatrix}.$$

Hence the motion equation for $W^{0,}_\mu$ is the following:

$$\begin{gathered}
\partial^\nu \partial_\nu W^{0,}_\mu = g_2^2 \left(W^{2,\nu} W^{2,}_\nu + W^{1,\nu} W^{1,}_\nu\right) W^{0,}_\mu - \\
-g_2^2 \left(W^{1,\nu} W^{1,}_\mu + W^{2,\nu} W^{2,}_\mu\right) W^{0,}_\nu + \\
+g_2 \partial^\nu \left(W^{1,}_\mu W^{2,}_\nu - W^{2,}_\mu W^{1,}_\nu\right) + \\
+g_2 \left(W^{1,\nu} \partial_\mu W^{2,}_\nu - W^{1,\nu} \partial_\nu W^{2,}_\mu - W^{2,\nu} \partial_\mu W^{1,}_\nu + W^{2,\nu} \partial_\nu W^{1,}_\mu\right) + \\
+\partial^\nu \partial_\mu W^{0,}_\nu .
\end{gathered} \tag{46}$$

$W^{1,}_\mu$ and $W^{2,}_\mu$ satisfy to similar equations.

This equation can be reformed as the following:

$$\begin{gathered}
\partial^\nu \partial_\nu W^{0,}_\mu = \left[g_2^2 \left(W^{2,\nu} W^{2,}_\nu + W^{1,\nu} W^{1,}_\nu + W^{0,\nu} W^{0,}_\nu\right)\right] \cdot W^{0,}_\mu - \\
-g_2^2 \left(W^{1,\nu} W^{1,}_\mu + W^{2,\nu} W^{2,}_\mu + W^{0,\nu} W^{0,}_\mu\right) W^{0,}_\nu + \\
+g_2 \partial^\nu \left(W^{1,}_\mu W^{2,}_\nu - W^{2,}_\mu W^{1,}_\nu\right) + \\
+g_2 \left(W^{1,\nu} \partial_\mu W^{2,}_\nu - W^{1,\nu} \partial_\nu W^{2,}_\mu - W^{2,\nu} \partial_\mu W^{1,}_\nu + W^{2,\nu} \partial_\nu W^{1,}_\mu\right) + \\
+\partial^\nu \partial_\mu W^{0,}_\nu .
\end{gathered}$$

This equation looks like to the Klein-Gordon equation of field $W_{\mu}^{0,}$ with mass

$$g_2\left[-\left(W^{2,\nu}W_{\nu}^{2,}+W^{1,\nu}W_{\nu}^{1,}+W^{0,\nu}W_{\nu}^{0,}\right)\right]^{\frac{1}{2}}. \tag{47}$$

and with the additional terms of the $W_{\mu}^{0,}$ interactions with others components of $\mathbf{W}$.

"Mass" (47) is invariant for the following transformations:

$$\left\{\begin{array}{l} W_r^{k,\prime}=W_r^{k,}\cos\lambda-W_s^{k,}\sin\lambda. \\ W_s^{k,\prime}=W_r^{k,}\sin\lambda+W_s^{k,}\cos\lambda; \end{array}\right|$$

$$\left\{\begin{array}{l} W_0^{k,\prime}=W_0^{k,}\cosh\lambda-W_s^{k,}\sinh\lambda, \\ W_s^{k,\prime}=W_s^{k,}\cosh\lambda-W_0^{k,}\sinh\lambda \end{array}\right|$$

with a real number λ, and $r\in\{1,2,3\}$, and $s\in\{1,2,3\}$,

and (47) is invariant for a global weak isospin transformation U':

$$W_{\nu}^{\prime}\rightarrow W_{\nu}^{\prime\prime}=U'W_{\nu}U'^{\dagger}$$

but is not invariant for a local transformation (45)

Equation (46) can be simplified as follows:

$$\begin{gathered}\sum_{\nu}g_{\nu,\nu}\partial^{\nu}\partial_{\nu}W_{\mu}^{0,}=\left[g_2^2\sum_{\nu\neq\mu}g_{\nu,\nu}\left((W_{\nu}^{2,})^2+(W_{\nu}^{1,})^2\right)\right]\cdot W_{\mu}^{0,}- \\ -g_2^2\sum_{\nu\neq\mu}g_{\nu,\nu}\left(W^{1,\nu}W_{\mu}^{1,}+W^{2,\nu}W_{\mu}^{2,}\right)W_{\nu}^{0,}- \\ +g_2\sum_{\nu}g_{\nu,\nu}\partial^{\nu}\left(W_{\mu}^{1,}W_{\nu}^{2,}-W_{\mu}^{2,}W_{\nu}^{1,}\right)+ \\ +g_2\sum_{\nu}g_{\nu,\nu}\left(W^{1,\nu}\partial_{\mu}W_{\nu}^{2,}-W^{1,\nu}\partial_{\nu}W_{\mu}^{2,}-W^{2,\nu}\partial_{\mu}W_{\nu}^{1,}+W^{2,\nu}\partial_{\nu}W_{\mu}^{1,}\right)+ \\ +\partial_{\mu}\sum_{\nu}g_{\nu,\nu}\partial^{\nu}W_{\nu}^{0,}.\end{gathered}$$

(here no of summation over indexes $^{\nu}_{\nu}$; the summation is expressed by $\sum$) with $g_{0,0}=1$, $g_{1,1}=g_{2,2}=g_{3,3}=-1$.

In this equation the form

$$g_2\left[-\sum_{\nu\neq\mu}g_{\nu,\nu}\left(\left(W_{\nu}^{2,}\right)^2+\left(W_{\nu}^{1,}\right)^2\right)\right]^{\frac{1}{2}}$$

varies in space, but it does not contain $W_{\mu}^{0,}$ and locally acts as mass - i.e. it does not allow to particles of this field to behave as a massless ones.

Let

$$\begin{array}{c} \alpha \stackrel{Def}{=} \arctan \frac{g_1}{g_2}, \\ Z_\mu \stackrel{Def}{=} \left(W_\mu^{0,} \cos\alpha - B_\mu \sin\alpha\right), \\ A_\mu \stackrel{Def}{=} \left(B_\mu \cos\alpha + W_\mu^{0,} \sin\alpha\right). \end{array}$$

In that case:

$$\sum_\nu g_{\nu,\nu}\partial_\nu\partial_\nu W_\mu^{0,} = \cos\alpha \cdot \sum_\nu g_{\nu,\nu}\partial_\nu\partial_\nu Z_\mu + \sin\alpha \cdot \sum_\nu g_{\nu,\nu}\partial_\nu\partial_\nu A_\mu.$$

If

$$\sum_\nu g_{\nu,\nu}\partial_\nu\partial_\nu A_\mu = 0$$

then

$$m_Z = \frac{m_W}{\cos\alpha}$$

with m_W from (47).

4.4 Rotations of the Cartesian frame and quarrks

Let:

$$\mathbf{e}_1' = \cos(\alpha)\,\mathbf{e}_1 - \sin(\alpha)\,\mathbf{e}_2; \mathbf{e}_2' = \sin(\alpha)\,\mathbf{e}_1 + \cos(\alpha)\,\mathbf{e}_2, \tag{48}$$

that is if

$$\begin{array}{rcl} \mathbf{x} & = & x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3 \\ & & \text{and} \\ \mathbf{x} & = & x_1'\mathbf{e}_1' + x_2'\mathbf{e}_2' + x_3'\mathbf{e}_3' \end{array}$$

then

$$\begin{array}{r} x_1' = x_1\cos(\alpha) - x_2\sin(\alpha); \\ x_2' = x_1\sin(\alpha) + x_2\cos(\alpha); \\ x_3' = x_3; \end{array}$$

and for any function φ:

$$\begin{aligned}
\partial_1'\varphi &= \left(\partial_1\varphi\cdot\cos\alpha - \partial_2\varphi\cdot\sin\alpha\right);\\
\partial_2'\varphi &= \left(\partial_2\varphi\cdot\cos\alpha + \partial_1\varphi\cdot\sin\alpha\right);\\
\partial_3'\varphi &= \partial_3\varphi.
\end{aligned} \tag{49}$$

From (8):

$$\begin{aligned}
j_1' = -\varphi^\dagger\left(\beta^{[1]}\cos\left(\alpha\right) - \beta^{[2]}\sin\left(\alpha\right)\right)\varphi;\\
j_2' = -\varphi^\dagger\left(\beta^{[1]}\sin\left(\alpha\right) + \beta^{[2]}\cos\left(\alpha\right)\right)\varphi;\\
j_3' = -\varphi^\dagger\beta^{[3]}\varphi.
\end{aligned}$$

Hence if for φ':

$$\begin{aligned}
j_1' &= -\varphi'^\dagger\beta^{[1]}\varphi';\\
j_2' &= -\varphi'^\dagger\beta^{[2]}\varphi';\\
j_3' &= -\varphi'^\dagger\beta^{[3]}\varphi'
\end{aligned}$$

and

$$\varphi' = U_{1,2}\left(\alpha\right)\varphi$$

then

$$\begin{aligned}
U_{1,2}^\dagger\left(\alpha\right)\beta^{[1]}U_{1,2}\left(\alpha\right) &= \beta^{[1]}\cos\alpha - \beta^{[2]}\sin\alpha;\\
U_{1,2}^\dagger\left(\alpha\right)\beta^{[2]}U_{1,2}\left(\alpha\right) &= \beta^{[2]}\cos\alpha + \beta^{[1]}\sin\alpha;\\
U_{1,2}^\dagger\left(\alpha\right)\beta^{[3]}U_{1,2}\left(\alpha\right) &= \beta^{[3]};
\end{aligned} \tag{50}$$

from (5): because

$$\rho = \varphi^\dagger\varphi = \varphi'^\dagger\varphi'$$

then

$$U_{1,2}^{\dagger}(\alpha)\,U_{1,2}(\alpha)=1_4. \tag{51}$$

If

$$U_{1,2}(\alpha)=\cos\frac{\alpha}{2}\cdot 1_4-\sin\frac{\alpha}{2}\cdot\beta^{[1]}\beta^{[2]}$$

then $U_{1,2}(\alpha)$ fulfils to all these conditions ((50), (51)). Moreover:

$$\begin{aligned}U_{1,2}^{\dagger}(\alpha)\,\beta^{[4]}U_{1,2}(\alpha)&=\beta^{[4]};\\ U_{1,2}^{\dagger}(\alpha)\,\gamma^{[0]}U_{1,2}(\alpha)&=\gamma^{[0]};\end{aligned} \tag{52}$$

and

$$U_{1,2}^{\dagger}(\alpha)\,\gamma^{[5]}U_{1,2}(\alpha)=\gamma^{[5]}.$$

Let $\widehat{H}_l'$ be the result of substitution $\beta^{[k]}$ by $\beta^{[k]\prime}=U_{1,2}^{\dagger}(\alpha)\,\beta^{[k]}U_{1,2}(\alpha)$ and ∂_k by $\partial_k'=\frac{\partial}{\partial x_k'}$ in $\widehat{H}_l$.

From (49), (50), (51) and (52):

$$\widehat{H}_l'=\mathrm{i}\left(\begin{array}{c}\beta^{[1]}\left(\begin{array}{c}\partial_1+\mathrm{i}\left(\Theta_1'\cos(\alpha)+\Theta_2'\sin(\alpha)\right)+\\ +\mathrm{i}\left(\Upsilon_1'\cos(\alpha)+\Upsilon_2'\sin(\alpha)\right)\gamma^{[5]}\end{array}\right)+\\ +\beta^{[2]}\left(\begin{array}{c}\partial_2+\mathrm{i}\left(-\Theta_1'\sin(\alpha)+\Theta_2'\cos(\alpha)\right)+\\ +\mathrm{i}\left(-\Upsilon_1'\sin(\alpha)+\mathrm{i}_2'\cos(\alpha)\right)\gamma^{[5]}\end{array}\right)+\\ +\beta^{[3]}\left(\partial_3+\mathrm{i}\Theta_3'+\mathrm{i}\Upsilon_3'\gamma^{[5]}\right)+\\ +\mathrm{i}M_0'\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}.\end{array}\right)$$

Therefore if

$$\begin{aligned}\Theta_0'&=\Theta_0,\\ \Theta_1'&=\Theta_1\cos(\alpha)-\Theta_2\sin(\alpha),\\ \Theta_2'&=\Theta_1\sin(\alpha)+\Theta_2\cos(\alpha),\\ \Theta_3'&=\Theta_3\end{aligned}$$

and the same formulas hold for $\langle\Upsilon_0,\Upsilon_1,\Upsilon_2,\Upsilon_3\rangle$ then $\widehat{H}_l'=\widehat{H}_l$ for the Cartesian frame reference rotation (48).

But:

$$U_{1,2}^{\dagger}(\alpha)\,\zeta^{[1]}U_{1,2}(\alpha)=\zeta^{[1]}\cos\alpha-\eta^{[2]}\sin\alpha;$$
$$U_{1,2}^{\dagger}(\alpha)\,\eta^{[2]}U_{1,2}(\alpha)=\eta^{[2]}\cos\alpha+\zeta^{[1]}\sin\alpha; \quad (53)$$

$$U_{1,2}^{\dagger}(\alpha)\,\zeta^{[2]}U_{1,2}(\alpha)=\zeta^{[2]}\cos\alpha-\eta^{[1]}\sin\alpha;$$
$$U_{1,2}^{\dagger}(\alpha)\,\eta^{[1]}U_{1,2}(\alpha)=\eta^{[1]}\cos\alpha+\zeta^{[2]}\sin\alpha; \quad (54)$$

$$U_{1,2}^{\dagger}(\alpha)\,\zeta^{[3]}U_{1,2}(\alpha)=\zeta^{[3]};$$
$$U_{1,2}^{\dagger}(\alpha)\,\eta^{[3]}U_{1,2}(\alpha)=\eta^{[3]};$$

$$U_{1,2}^{\dagger}(\alpha)\,\gamma_{\zeta}^{[0]}U_{1,2}(\alpha)=\gamma_{\zeta}^{[0]}\cos\alpha-\gamma_{\eta}^{[0]}\sin\alpha;$$
$$U_{1,2}^{\dagger}(\alpha)\,\gamma_{\eta}^{[0]}U_{1,2}(\alpha)=\gamma_{\eta}^{[0]}\cos\alpha+\gamma_{\zeta}^{[0]}\sin\alpha; \quad (55)$$
$$U_{1,2}^{\dagger}(\alpha)\,\gamma_{\theta}^{[0]}U_{1,2}(\alpha)=\gamma_{\theta}^{[0]};$$

$$U_{1,2}^{\dagger}(\alpha)\,\zeta^{[4]}U_{1,2}(\alpha)=\zeta^{[4]}\cos\alpha+\eta^{[4]}\sin\alpha;$$
$$U_{1,2}^{\dagger}(\alpha)\,\eta^{[4]}U_{1,2}(\alpha)=\eta^{[4]}\cos\alpha-\zeta^{[4]}\sin\alpha; \quad (56)$$
$$U_{1,2}^{\dagger}(\alpha)\,\theta^{[4]}U_{1,2}(\alpha)=\theta^{[4]}.$$

Terefore from (53), (54), (55), (56):

$\widehat{H}(\zeta)$ is mixed with $\widehat{H}(\eta)$ for this rotation. For other rotations of the frame reference: the leptonn hamiltonians do persist, but the chromatic hamiltonians are mixed with other chromatic hamiltonians.

Therefore the chromatic triplet elements can not be separated in the space. These elements must be localized in the identical place.

Each chromatic pentad contains two mass elements. Hence, every family contains two sorts of the chromatic particles of tree colors. I call these particles as *quarrks.*

5 The two-event case

Let $N^{\text{`}} = 2$ and

$$\int_{D_1} d^3\mathbf{x}^{(1)} \int_{D_2} d^3\mathbf{x}^{(2)} \cdot \rho\left(t, \mathbf{x},\ \mathbf{y}\right) \overset{Def}{=} \mathbf{P}\left(\exists \mathbf{x} \in D_1 : \exists \mathbf{y} \in D_2 : \alpha_{1,2}\left(t, \mathbf{x}, \mathbf{y}\right)\right)$$

Complex functions $\varphi_{j_1,j_2}\left(t, \mathbf{x}, \mathbf{y}\right)$ ($j_k \in \{1,2,3,4\}$) exist for which:

$$\rho\left(t, \mathbf{x},\ \mathbf{y}\right) = 4 \sum_{j_1=1}^{4} \sum_{j_2=1}^{4} \varphi^{*}_{j_1,j_2}\left(t, \mathbf{x}, \mathbf{y}\right) \varphi_{j_1,j_2}\left(t, \mathbf{x}, \mathbf{y}\right).$$

If

$$\Psi\left(t, \mathbf{x}, \mathbf{y}\right) \overset{Def}{=}$$

$$= \sum_{j_1=1}^{4} \sum_{j_2=1}^{4} \varphi_{j_1,j_2}\left(t, \mathbf{x}, \mathbf{y}\right) \left(\psi^{(1)\dagger}_{j_1}\left(\mathbf{x}\right) \psi^{(2)\dagger}_{j_2}\left(\mathbf{y}\right) - \psi^{(2)\dagger}_{j_2}\left(\mathbf{y}\right) \psi^{(1)\dagger}_{j_1}\left(\mathbf{x}\right)\right) \Phi_0$$

then

$$\Psi^{\dagger}\left(t, \mathbf{x}, \mathbf{y}\right) \Psi\left(t, \mathbf{x}, \mathbf{y}\right) =$$

$$= 4 \sum_{j_1=1}^{4} \sum_{j_2=1}^{4} \varphi^{(1,2)*}_{j_1,j_2}\left(t, \mathbf{x}', \mathbf{y}'\right) \varphi^{(1,2)}_{j_1,j_2}\left(t, \mathbf{x}, \mathbf{y}\right) \cdot \delta\left(\mathbf{x} - \mathbf{x}'\right) \delta\left(\mathbf{y} - \mathbf{y}'\right).$$

Like to (10): the system with unknown complex functions $Q^{(1)}_{j_1,k_1}$, $Q^{(2)}_{j_2,k_2}$, $Q^{(1,2)}_{j_1,k_1;j_2,k_2}$:

$$\left\{ \begin{array}{c} \sum_{k_1=1}^{4} Q^{(1)}_{j_1,k_1} \varphi_{k_1,j_2} + \sum_{k_2=1}^{4} Q^{(2)}_{j_2,k_2} \varphi_{j_1,k_2} + \sum_{k_1=1}^{4} \sum_{k_2=1}^{4} Q^{(1,2)}_{j_1,k_1;j_2,k_2} \varphi_{k_1,k_2} = \\ = \partial_t \varphi_{j_1,j_2} - \sum_{r=1}^{3} \left(\sum_{k_1=1}^{4} \beta^{[r]}_{j_1,k_1} \frac{\partial}{\partial x_r} \varphi_{k_1,j_2} + \sum_{k_2=1}^{4} \beta^{[r]}_{j_2,k_2} \frac{\partial}{\partial y_r} \varphi_{j_1,k_2}\right), \\ Q^{(1)*}_{k_1,j_1} = -Q^{(1)}_{j_1,k_1}, \\ Q^{(2)*}_{k_2,j_2} = -Q^{(2)}_{j_2,k_2}, \\ Q^{(1,2)*}_{k_1,j_1;j_2,k_2} = -Q^{(1,2)}_{j_1,k_1;j_2,k_2}, \\ Q^{(1,2)*}_{j_1,k_1;k_2,j_2} = -Q^{(1,2)}_{j_1,k_1;j_2,k_2} \end{array} \right|$$

has got a solution.
et cetera...

6 Conclusion

Therefore all physical events are interpreted by well-known elementary particles - leptons, quarks and gauge bosons. And if anybody will claim that he has found Higgs then not believe - it is not Higgs.

Appendix

Three *chromatic* pentads: *red pentad* ζ:

$$\zeta^{[1]} \stackrel{Def}{=} \begin{bmatrix} -\sigma_1 & 0_2 \\ 0_2 & \sigma_1 \end{bmatrix}, \zeta^{[2]} \stackrel{Def}{=} \begin{bmatrix} -\sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \zeta^{[3]} \stackrel{Def}{=} \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & \sigma_3 \end{bmatrix},$$

$$\gamma_\zeta^{[0]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & -\sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}, \zeta^{[4]} \stackrel{Def}{=} -\mathrm{i} \cdot \begin{bmatrix} 0_2 & \sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}$$

green pentad η:

$$\eta^{[1]} \stackrel{Def}{=} \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & \sigma_1 \end{bmatrix}, \eta^{[2]} \stackrel{Def}{=} \begin{bmatrix} -\sigma_2 & 0_2 \\ 0_2 & \sigma_2 \end{bmatrix}, \eta^{[3]} \stackrel{Def}{=} \begin{bmatrix} -\sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix},$$

$$\gamma_\eta^{[0]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & -\sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}, \eta^{[4]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & \sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix},$$

blue pentad θ:

$$\theta^{[1]} \stackrel{Def}{=} \begin{bmatrix} -\sigma_1 & 0_2 \\ 0_2 & \sigma{-_1} \end{bmatrix}, \theta^{[2]} \stackrel{Def}{=} \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & \sigma_2 \end{bmatrix}, \theta^{[3]} \stackrel{Def}{=} \begin{bmatrix} -\sigma_3 & 0_2 \\ 0_2 & \sigma_3 \end{bmatrix},$$

$$\gamma_\theta^{[0]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & -\sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix}, \theta^{[4]} \stackrel{Def}{=} -\mathrm{i} \cdot \begin{bmatrix} 0_2 & \sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix};$$

one *light pentad* β:

$$\beta^{[1]} \stackrel{Def}{=} \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \beta^{[2]} \stackrel{Def}{=} \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \beta^{[3]} \stackrel{Def}{=} \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix},$$

$$\gamma^{[0]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}, \beta^{[4]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}$$

and two *taste pentads*:
sweet pentad $\underline{\Delta}$:

$$\underline{\Delta}^{[1]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & -\sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}, \underline{\Delta}^{[2]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & -\sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}, \underline{\Delta}^{[3]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & -\sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix},$$

$$\underline{\Delta}^{[0]} \stackrel{Def}{=} \begin{bmatrix} -1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \underline{\Delta}^{[4]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix},$$

bitter pentad $\underline{\Gamma}$:

$$\underline{\Gamma}^{[1]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & -\sigma_1 \\ \sigma_1 & 0_2 \end{bmatrix}, \underline{\Gamma}^{[2]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & -\sigma_2 \\ \sigma_2 & 0_2 \end{bmatrix}, \underline{\Gamma}^{[3]} \stackrel{Def}{=} \mathrm{i} \cdot \begin{bmatrix} 0_2 & -\sigma_3 \\ \sigma_3 & 0_2 \end{bmatrix},$$

$$\underline{\Gamma}^{[0]} \stackrel{Def}{=} \begin{bmatrix} -1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \underline{\Gamma}^{[4]} \stackrel{Def}{=} \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}.$$

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